

Adiabatic Expansion of Soda Pop—C.E. Mungan, Summer 2000

Demo 15-04 in the Video Encyclopedia of Physics Demonstrations presents a plot of the temperature of the air in a bottle as it is pressurized to the point that it blows its top. The temperature just before blowing is $T_i = 306$ K. After blowing, the temperature levels off in about 10 seconds to $T_f = 300$ K. What is the gauge pressure inside the bottle just before blowing?

If $p_i > p_{atm}$ and $p_f = p_{atm}$ are the initial and final pressures, and $V_i = V_{bottle}$ and $V_f > V_{bottle}$ the corresponding volumes of the air, then

$$p_i V_i = nRT_i \tag{1a}$$

and

$$p_f V_f = nRT_f \tag{1b}$$

since air is very nearly ideal at these temperatures and pressures. Dividing Eq. (1b) by (1a) gives

$$\frac{V_f}{V_i} = \frac{p_i}{p_f} \cdot \frac{T_f}{T_i}. \tag{2}$$

Furthermore, the 10-second blow-off time is sufficiently short that no significant heat exchange between the bottled air and the surroundings can occur. Thus, the expansion is adiabatic so that

$$p_i V_i^\gamma = p_f V_f^\gamma \Rightarrow \frac{V_f}{V_i} = \left(\frac{p_i}{p_f} \right)^{1/\gamma} \tag{3}$$

where $\gamma = 7/5$ since air is mainly a mixture of diatomic gases. Equating the right-hand sides of Eqs. (2) and (3) eliminates the unknown volumes. The result can be rearranged into the form

$$p_i = p_f \left(\frac{T_f}{T_i} \right)^{\frac{\gamma}{\gamma-1}}. \tag{4}$$

Using the binomial expansion, the initial gauge pressure is thus

$$\frac{p_{gauge}}{p_{atm}} \cong \left(\frac{T_i}{T_f} - 1 \right) \left(1 + \frac{\nu}{2} \right) \tag{5}$$

where the number of degrees of freedom per molecule for a diatomic well below the vibrational excitation temperature is $\nu = 5$. Using the given data, the result is 7%.

Suppose that the water vapor inside the bottle is saturated, as it would be inside a soda bottle which has had plenty of time to reach equilibrium before it is opened. Then it is not hard to show that the ratio of the changes in the pressure and temperature of the water vapor during the adiabatic expansion in the absence of condensation is

$$\frac{\Delta p}{\Delta T} \cong \left(\frac{p_{vapor}}{T_{room}} \right) \left(1 + \frac{\nu}{2} \right) \tag{6}$$

where now $\nu = 6$ since water has three rotational degrees of freedom. From Fig. 12.34 in Cutnell and Johnson, we see that $p_{\text{vapor}} = 4.8 \text{ kPa}$ for $T_{\text{room}} = 32 \text{ }^\circ\text{C}$, so that Eq. (6) equals 63 Pa/K . In contrast, the slope of the vaporization curve is seen from this same figure to be approximately $(4.8 - 2.5) \text{ kPa} / (32 - 21) \text{ }^\circ\text{C} = 210 \text{ Pa/K}$. (This is in reasonable agreement with the Clausius-Clapeyron equation in the form $Lp_{\text{vapor}} / RT_{\text{room}}^2$ where $L = 40.7 \text{ kJ/mol}$ is the latent heat of vaporization of water—see Stowe p. 398.) Some water thus immediately condenses so that the partial pressure of the vapor falls more steeply and remains on the vaporization curve. This is one reason why a mist appears at the top of a freshly opened soda. (Undoubtedly another reason has to do with carbon dioxide rapidly leaving the liquid and effervescently entraining droplets with it.)