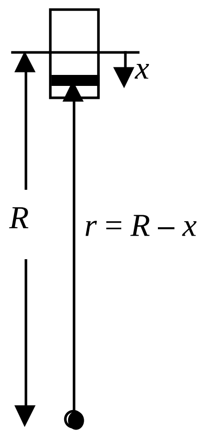
The Black Hole Shredder—C.E. Mungan, Spring 2001

John Denker asked the following amusing little question on PHYS-L. What mass would a black hole have to have in order to shred a brick before it winks out of sight, i.e., if the tidal stresses just manage to pull the brick apart at the event horizon?

Let the black hole have mass M and the brick have length L, cross-sectional area A, and mass m. Neglect the small change in the density $\rho = m / AL$ of the brick during its approach to the hole. The maximum stress is applied to the brick when it is oriented with its length in the radial direction. Let R be the distance from the center of the black hole to the center of the brick.

First let's calculate the free-fall acceleration, a, of the brick assuming it remains intact.



Applying Newton's second law to the entire brick, we get

$$ma = F_{net} = \int dw = \int g(r)dm \tag{1}$$

where the only external force is the weight w of the brick. Since $g(r) = GM / r^2$, this becomes

$$a = \bar{g} = \frac{\int_{R-L/2}^{R+L/2} g(r)\rho A dr}{\rho A L} = \frac{\left.\frac{-GM}{r}\right|_{R-L/2}^{R+L/2}}{L} = \frac{\frac{GM}{R-L/2} - \frac{GM}{R+L/2}}{L} \approx \frac{GM}{R^2} \tag{2}$$

assuming $L << R$. As one might have guessed, gravity acts at approximately the center of mass owing to the small size of the brick.

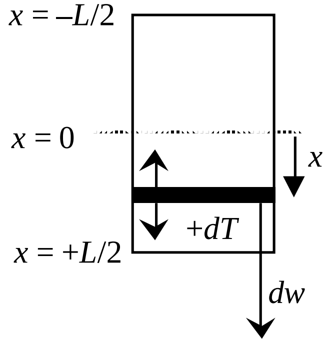
Now apply Newton's second law to a small slice of the brick, as sketched at the top of the following page. Denoting tension by T,

$$(\rho A dx)g(x) + dT(x) = (\rho A dx)a \tag{3}$$

so that

$$dT = (a-g)\rho A dx = \left[\frac{GM}{R^2} - \frac{GM}{(R-x)^2}\right]\rho A dx \approx -\frac{2x}{R} a \rho A dx\,. \tag{4}$$

The negative sign means that the upward tension on the indicated slice is greater than the downward tension, which restrains it from accelerating with the local free-fall value of g.



Integrating Eq. (4) and dividing by the cross-sectional area gives the stress profile in the brick,

$$\sigma(x) = \frac{GM\rho}{R^3}\left(\frac{L^2}{4} - x^2\right) \tag{5}$$

since $T = 0$ at the free ends. The maximum stress thus occurs at the center. The brick will be on the verge of crumbling when this equals the tensile strength,

$$\sigma_{\max} = \frac{GM\rho}{R^3}\frac{L^2}{4}. \tag{6}$$

Finally, we set R equal to the Schwarzschild radius. A particle of mass m can just escape from an astronomical body of mass M and radius R if its total energy is zero so that it reaches infinity with zero speed,

$$\frac{1}{2}m\upsilon_{esc}^2 - \frac{GMm}{R} = 0. \tag{7}$$

Setting the escape speed equal to the speed of light, $\upsilon_{esc} = c$, so that not even light can escape, gives the Schwarzschild radius as

$$R = \frac{2GM}{c^2}. \tag{8}$$

Substituting Eq. (8) into (6) and solving for the required mass of the black hole gives

$$\boxed{M = \frac{Lc^3}{G}\sqrt{\frac{\rho}{32\sigma_{\max}}}}. \tag{9}$$

I don't happen to have an ordinary building brick (ASTM C62) lying around to make measurements of its size and mass, but according to the internet it is 9" = 23 cm long and has a density of 120 lb/ft^3 = 1900 kg/m^3. Nelson Physics <http://www.nelsonitp.com/physics/> cites a tensile strength of 1 MPa (which seems a bit high but will therefore give a lower bound on *M*). The resulting value for *M* is 360 solar masses! Needless to say, no ordinary black hole formed during a stellar collapse could have a mass this large, although the supermassive holes postulated to exist at the center of galaxies such as the Milky Way certainly can.