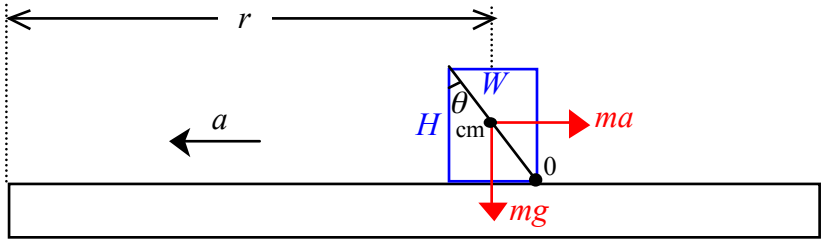
Box Slipping vs Tilting in an Accelerating Frame—C.E. Mungan, Summer 2020

A box of height H and width W is on the back of a flatbed truck accelerating to the left, as shown in the following sketch. If the coefficient of static friction between the box and bed is μ, which will happen first if the magnitude of the acceleration a is gradually increased: will the box slip rightward or will it tilt backward around point 0?



If the box slips, the friction force must be at its largest value μN where the normal force balances gravity vertically so that $N = mg$. The frictional force must provide the horizontal acceleration of the box just before slipping occurs, such that $\mu mg = ma$. Therefore the box slips when

$$\frac{a}{g} > \mu . \tag{1}$$

Once the box starts to slide, it will continue to do so until it falls off the end of the flatbed (assuming there is no end gate in the truck to restrain it).

On the other hand, the box starts to tilt if the torques about 0 balance in the accelerating frame of reference of the box. In that frame, there is a pseudoforce ma acting backward through the center of mass (cm), as indicated by the red arrow in the diagram above. The normal and frictional forces act through 0 at the instant of tilting and thus produce no torque about that point. Thus the torque due to the pseudoforce must balance the torque due to the gravitational force, indicated by the other red arrow above, so that $maD\cos\theta = mgD\sin\theta$ where D is the distance between 0 and the cm, and θ is the angle labeled in the preceding diagram. Hence the box begins to tilt backward when

$$\frac{a}{g} > \tan\theta = \frac{W}{H} . \tag{2}$$

Once the box starts tilting back, the gravitational moment arm decreases whereas the pseudoforce moment arm increases (because D starts becoming more nearly vertical). As a result, once the box starts tilting, it will continue to do so and immediately fall over onto side H. If the coefficient of restitution is small enough to dissipate little of the mechanical energy during the collisions between the box and the bed, then the box will start tumbling end over end until it falls off the back of the truck.

To summarize, whether the box slips or tumbles depends on which is smaller, μ or W / H. A variation on this problem was suggested to me by Stephen Irons. Instead of translational acceleration, suppose the box is on a turntable rotating about the left edge at angular speed ω in the diagram above. If $W << r$, then we can treat the acceleration of all parts of the box as being the same and given by $a = \omega^2 r$.