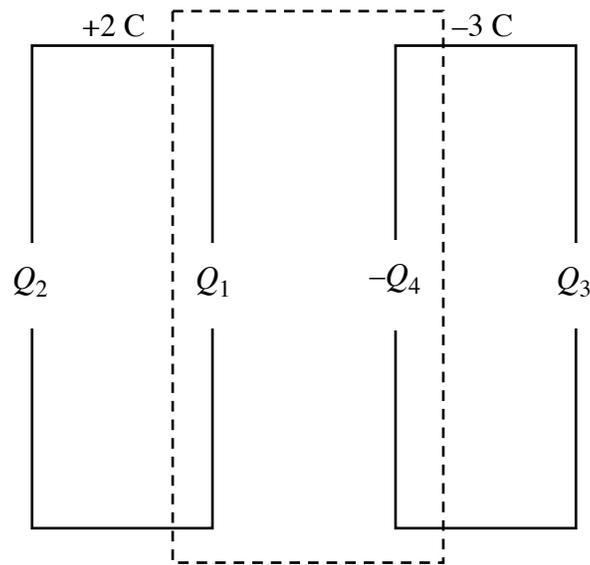


Charged Parallel Metal Plates—C.E. Mungan, Spring 2014

Suppose there are two large parallel metal plates. To avoid the problem of infinities, let us say each plate is 1 mm thick, each plate is square of length 1 m on a side, and the two plates are separated by 1 mm. A net charge of +2 C is put on the left plate and charge -3 C on the right plate. Neglecting the fringing, what are the electric fields to the left of the two plates, in the gap between the two plates, and to the right of the two plates? Spell out all physical assumptions used in your solution and not just the correct numerical answers.

First I give the answers, with reference to the following figure showing the charges on the inner and outer surfaces of each plate.



We require

$$Q_2 + Q_1 = 2 \text{ C} \quad (1)$$

and

$$Q_3 - Q_4 = -3 \text{ C} \quad (2)$$

and

$$Q_4 = Q_1 \quad (3)$$

and finally

$$Q_3 = Q_2 \quad (4)$$

whose simultaneous solution is $Q_1 = Q_4 = 2.5$ C and $Q_2 = Q_3 = -0.5$ C. Therefore to the left of the plates the electric field is $E = 5.65 \times 10^{10}$ N/C pointing rightward, between the two plates $E = 2.82 \times 10^{11}$ N/C pointing rightward, and to the right of the plates $E = 5.65 \times 10^{10}$ N/C pointing leftward.

Here are the four assumptions that went into my answer:

- (i) charge is conserved;
- (ii) an infinite plane of charge symmetrically creates equal magnitude fields at equal distances left or right of the plane, and those fields are directed normally to the plane;
- (iii) electric fields linearly superpose vectorially;
- (iv) the static electric field inside a metal is zero.

Equations (1) and (2) are explicitly written down to satisfy assumption (i).

To obtain Eq. (3), apply Gauss' law to the dashed gaussian surface in the preceding figure, by making use of assumption (iv) to show that the net flux on the surface is zero.

Combining Gauss' law with assumption (ii) gives the standard formula

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \hat{n} \quad (5)$$

for a single infinite plane of uniform charge (suspended in free space without the presence of a metal sheet).

Finally, use assumption (iii) to superpose the fields created by all four planes of charge at an arbitrary point inside either the left or right metal plate. Equation (4) must hold, in order to satisfy assumption (iv) at that point. I see no way to prove Eq. (4) directly from Gauss' law by any clever choice of gaussian surface.

We have now justified Eqs. (1) to (4), and therefore their simultaneous solutions for the charges on each of the four surfaces. Finally, if we superpose the fields created by those charges using Eq. (5), or if we draw gaussian rectangular surfaces that surround one plane of charge at a time and an adjacent region of air, we obtain the previously stated electric field vectors for the three regions of interest.