Coins Puzzle—C.E. Mungan, Fall 2005

While blindfolded, you are presented with a tray of identical coins. You are told that N of them are heads up. How can you divide them into two groups such that each group ends up with the same number of heads? You may manipulate the coins any way you like, but have no means of determining which are heads and which are tails.

Although this puzzle may initially seem impossible, the solution is both simple to explain and to prove. Simply place any N coins into one group and the remaining coins into the other group. Now flip over over all N coins in the first group and the task is accomplished!

Proof: The first group of N coins has say H number of heads and thus $N–H$ tails initially. Since there are N heads initially in all, the second group must contain $N–H$ heads. Now when we flip over every coin in the first group, its $N–H$ tails all become heads (and all its original tails become heads). Thus each group ends up with $N–H$ heads. QED

Example: Suppose you start out with 10 coins, of which 3 are heads up. You randomly form a group of three (say it contains 1 head and 2 tails), and another group of seven (which therefore contains 2 heads and 5 tails). When we flip over every coin in the first pile, it becomes a group of 1 tail and 2 heads. Thus both groups now contain 2 heads.

There are several interesting observations one can make about this puzzle and its solution:

1. This example worked just fine even though N was odd. It also works fine if N is equal to zero or if it is equal to every coin on the tray.[1] No special cases need to be treated separately!

2. There are only two legal operations one can make and we used each of them exactly once: dividing the coins into groups of known numbers, and flipping coins over. While the first operation is obvious, the second is less so but can be discovered by considering the case of odd N. To motivate this realization, it is probably a good idea to explicitly suggest in the problem statement that one consider that case.

3. One does not of course know how many heads end up in each group, since H can vary randomly between[2] 0 and N. But the problem does not ask us to determine this and it isn't hard to see we can't possibly be expected to figure it out! Again, this is probably clear once one realizes one can flip coins over.

4. As with many problems of this sort, it is very helpful to first try to solve some simple case, such as a tray of 4 coins of which 2 are said to be heads. I recommend this as an initial strategy.

5. You do not need to count the total number of coins. That number could even be infinite (provided N is finite) and you can still effect the two required operations!

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[1] I am assuming a group of zero is permitted. If not, there is no solution if only one coin is on the tray, but if there is more than one coin, you can simply choose any one coin out of the final pile of tails and call it another group.

[2] Actually if $N > n/2$ then the lower limit on H is $2N–n$, which is greater than zero, where n is the total number of coins on the tray.

6. Finally, it is important to try to minimize the amount of time you waste trying to come up with illegal tricks to employ. The sooner you reconcile yourself to the fact that there is a legal and well-defined solution, the sooner you can get on with finding it. In my experience, this is the greatest psychological hurdle to solving such puzzles and is a great lesson for the real world. Creativity and persistence (without having to stoop to cheating) are useful skills!

7. As an extension of this problem, I have considered whether there are any strategies for maximizing the number of heads you are likely to end up with. Clearly the answer is yes if H is either zero or every coin on the tray. In the former case, start by flipping over every coin. Now in both cases, split the coins into two even piles. If the number of coins is odd, flip any one over and add it to either pile. In contrast, if you follow the algorithm in the above solution, you will end up with zero heads. However, I have not been able to find an alternate strategy for other possible values of H if you insist that you must still end up with an equal number of heads in both piles. Can you?