Theory of Holonomic Constraints—C.E. Mungan, Fall 2002

Suppose we have N particles in D dimensions with C constraints. Assume for simplicity that there are no nonconservative forces other than the constraint forces. Choose the generalized coordinates so that

$$q_j = 0 \quad \text{for } j = 1,\ldots,C \tag{1}$$

expresses the constraints. Let

$$L = K - U \tag{2}$$

where U includes only the real, conservative potentials.

The Lagrange equations for the unconstrained coordinates tell us the actual motion of the system,

$$\frac{d}{dt}\left[\frac{\partial L(\{q_j\}=0)}{\partial \dot{q}_k}\right] = \frac{\partial L(\{q_j\}=0)}{\partial q_k} \quad \text{for } k = C+1,\ldots,DN\,. \tag{3}$$

The notation $L(\{q_j\}=0)$ means that we impose the constraints (1) <u>before</u> taking the derivatives.

If all we wanted was to find the motion of the system, we are now done. However, if we wish, we can find the generalized, nonconservative, constraint forces $\mathcal{F}_j$ which make $q_j = 0$. (Since the constraints are holonomic, these nonconservative forces are nondissipative.) Solve the Lagrange equations for the constrained coordinates,

$$\mathcal{F}_j = \left[\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j}\right]_{\{q_j\}=0} \tag{4}$$

where the subscripted $\{q_j\}=0$ means that this time we impose the constraints (1) <u>after</u> taking the derivatives.