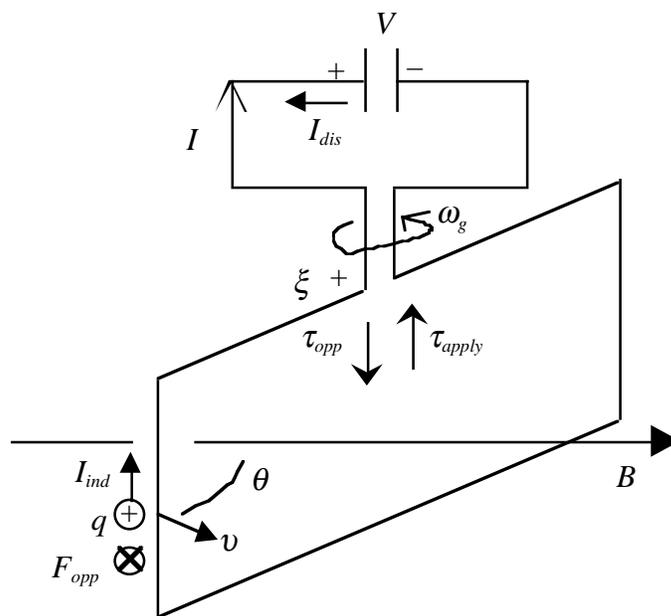


## A Generator and a Capacitor—C.E. Mungan, Fall 1999

A simple demo of the properties of capacitors and generators/motors was first shown to me by T.J. Miller and is also described by Robert Ehrlich in his lovely book of demos, *Why Toast Lands Jelly-Side Down: Zen and the Art of Physics Demonstrations*. A Pasco hand-cranked generator is attached to a 1-F capacitor. Crank the generator a few times to charge up the capacitor. (By the way, Ehrlich points out that you should not crank too rapidly because typical 1-F capacitors have a maximum voltage of merely 5 V.) Now stop turning the crank and then remove your hand from it. The capacitor will begin to discharge, reversing the direction of the current through the generator, which will consequently act as a motor and the hand crank will begin to turn. Surprisingly however, the crank will turn in the SAME direction as you originally turned it. Since the current reverses, why doesn't the direction of rotation of the crank reverse?

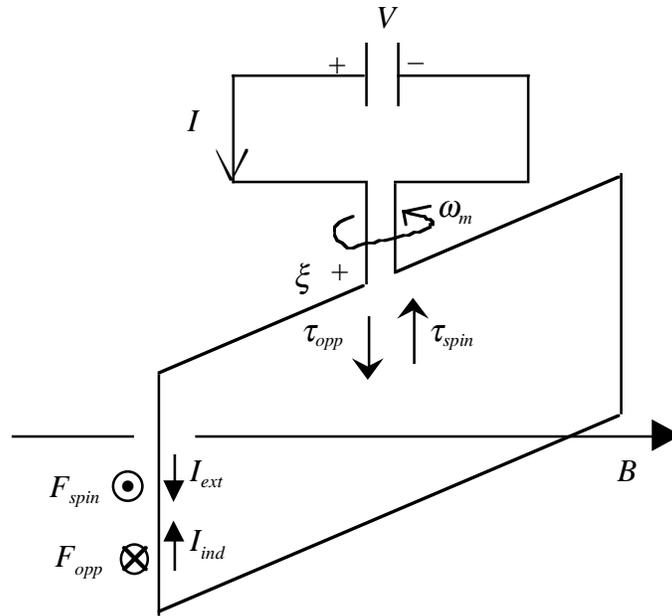
In my mind, the only convincing explanation is to actually draw the circuit diagrams for the two cases and apply the appropriate right-hand rules. For simplicity, I will represent the generator coil as having only a single loop and I will neglect mechanical friction in the generator, so that the only source of dissipation comes from the coil resistance  $R$ .



Start with the case of the generator as sketched above. Spin the coil at angular speed  $\omega_g$  by applying torque  $\tau_{apply}$  with your hand. Hence at the instant the coil has the indicated orientation, the front segment has velocity  $v$  which makes angle  $\theta$  with the permanent magnetic field  $B$ . Consider a fiducial positive charge  $q$  in this segment; it is driven upward by the  $qv \times \mathbf{B}$  magnetic force. Consequently, a current  $I_{ind}$  is generated, or equivalently an induced emf  $\xi$ , which charges the capacitor up to some potential difference  $V$  against its discharge current  $I_{dis}$ . (Although I have not drawn the split-ring commutator, it is understood there must be one to avoid reversing the sign of  $\xi$  every half-cycle.)

However, we now have a net current  $I = I_{ind} - I_{dis}$  perpendicular to a magnetic field  $B$ . As a result, there will be a force  $\mathbf{F}_{opp} = I\mathbf{L} \times \mathbf{B}$  on the coil (where  $L$  is the combined length of the front and back segments), or equivalently a torque  $\tau_{opp}$ , opposing your cranking. For example, if you turn the crank at constant angular speed, then  $\tau_{apply} = \tau_{opp}$ , i.e., you expend energy at the rate of  $\tau_{apply}\omega_g$  to charge up the capacitor.

Next consider the motor at the same instant of orientation as the generator as follows.



The potential difference  $V$  of the capacitor drives an external current  $I_{ext}$  into the coil, opposite in direction to the net current above. This current in the permanent field  $B$  gives rise to a force  $\mathbf{F}_{spin} = I_{ext} \mathbf{L} \times \mathbf{B}$ , and hence to a torque  $\tau_{spin}$ , which spins the coil at angular speed  $\omega_m$  in the same direction as above. However, this creates an opposing induced current  $I_{ind}$  by Lenz's law, driven by the induced ("back") emf  $\xi$  according to Ohm's law,  $I_{ind} = \xi / R$ . This results in an opposing force  $\mathbf{F}_{opp} = I_{ind} \mathbf{L} \times \mathbf{B}$  and hence torque  $\tau_{opp}$ . Assuming the capacitor discharges slowly enough that quasi-equilibrium is always maintained, then  $\tau_{spin} = \tau_{opp}$  and the coil rotates at nearly constant angular speed  $\omega_m$ .

This answers the question, but it would still be nice to have additional insight into why the capacitor charges in the first situation but discharges in the second situation, even though the crank is turning the same way in both cases. How does the device "know" that you have taken off your hand? If we apply Kirchhoff's Loop Rule to either circuit, we see that the net current is

$$I = \frac{V - \xi}{R},$$

which is Eq. (22.5) in Cutnell & Johnson. In the motor case, the applied voltage  $V$  exceeds the back emf  $\xi$  and so  $I$  is counter-clockwise, while in the generator case, the generated emf  $\xi$  exceeds the load voltage  $V$  so that  $I$  is instead clockwise. Now consider what would happen if  $\xi$  were maintained exactly equal to  $V$  at all times on average, so that  $I = 0$  and there is no net energy dissipation. But  $\xi$  is directly proportional to  $\omega$  via the generator equation,  $\xi = KB\omega$ , where  $K$  is called the motor constant and is the product of the effective area of the coil and an average over the sine of the angle between the armature and the field which depends on the specific design of the commutator—cf. Eq. (2) on page 986 of Tipler's *Physics for Scientists and Engineers*, 4<sup>th</sup> edition. Hence to do this, the rotation rate of the coil would have to be maintained at the value  $\omega_0 = V / KB$ . Clearly therefore  $\omega_g > \omega_0 > \omega_m$ . Now jump into a frame of reference rotating at angular speed  $\omega_0$  about the axis of the coil of the actual hand crank. Seen from this frame, the crank rotates in the forward direction for the case of the generator, but in the reverse direction for the case of the motor. Thus seen from the point of view of this equilibrium frame (viz. dissipationless when the coil is observed to be at rest), the direction of rotation of the crank DOES reverse when you remove your hand from it!