Faraday Isolators and Kirchhoff's Law: A Puzzle—C.E. Mungan

Statement of the Puzzle

A Faraday isolator is a standard optical component in many laser labs. It is a one-way light valve, permitting radiation to pass through one way and not the other, thus preventing harmful back-reflections from reaching a laser equipped with such an isolator. It consists of an input linear polarizer, a magnetic rotator based on the Faraday effect, and an output linear polarizer whose pass axis is rotated 45˚ relative to the input polarizer. The magnetic field is chosen for the wavelength of the laser light of interest so that the plane of polarization is rotated by 45˚ into the output polarizer's pass direction. In contrast to optical activity, the direction of rotation is *independent* of the direction of propagation of the light. This means that if linearly polarized light is retro-reflected through the magnetic medium, it will not "unwind" its direction of polarization back to its initial state; on the contrary, it will now be rotated by 90˚ and hence will be blocked by the input polarizer from getting back to the laser! This is sometimes called an optical diode (e.g. A.E. Siegman, *Lasers*, 1986, p. 535). For diagrams and additional explanation, consult any good book on lasers and optics.

With such an isolator, I could construct a one-way window such that I could see you but you not see me. This is disquieting because it is not evident how to save Kirchhoff's law of radiation (absorptivity equals emissivity) with such a gizmo. The arrangement I have in mind is simple. A black sample is inserted into a cavity having perfectly reflective walls. Then I make a small hole in the walls into which I place a Faraday isolator. Now what prevents the sample from radiating away energy continuously out of the cavity through this window and cooling down to absolute zero?

Let's dispense with a couple of objections off the bat:

(1) A Faraday isolator only works at one wavelength, whereas a sample's thermal spectrum is necessarily broadband. Maybe the non-design wavelengths will compensate for what happens at the design wavelength of the isolator.

Response: I can put an ideal filter on the sample side of the isolator. Such a filter perfectly transmits a narrow passband, which I of course choose to coincide with the isolator's design wavelength, and perfectly reflects all other wavelengths back to the sample. A perfect filter of this sort can be approximately realized experimentally and, more importantly, can be postulated without violating any law of physics relevant to the problem at hand. For example, such a filter is postulated to prove the spectral version of Kirchhoff's law (i.e., that the absorptivity of a sample is equal to its emissivity wavelength by wavelength and not merely when integrated over all frequencies)—see M. Goldstein and I.F. Goldstein, *The Refrigerator and the Universe*, 1993, p. 243.

(2) A Faraday isolator only transmits the linear polarization in its pass direction of propagation and none in the other direction. Won't the rejected polarizations be absorbed by the device, causing it to heat up and thus warm the sample?

Response: The magnetic medium merely rotates the light's polarization, so that it can be transparent with anti-reflection coatings on its end faces. The polarizers are assumed to be ideal, i.e., perfectly transmitting the pass polarization and perfectly reflecting the orthogonal polarization. Hence the Faraday isolator need not absorb any light at all. One way to approximately do this, for example, is to use Nicol prisms as the polarizers. A full description of these and related prisms can be found in E. Hecht, *Optics*, 3rd ed., p. 338. Briefly, it is made out of two pieces of a birefringent material cut and pasted back together with a transparent cement at an appropriate angle to the optic axis. The *o*-ray (say) is totally internally reflected at the cement interface, while the *e*-ray is (at least partly) transmitted because it has an index of refraction which more nearly matches that of the cement.

Okay, so you should now understand the puzzle! As you can see by reading the quote from Wood's optics text below, this puzzle stumped some great physicists. But actually the resolution is straightforward and provided in the second to last paragraph in Rayleigh's paper which is quoted after Wood. Can you figure it out before reading that far?

R.W. Wood, "Magnetic Rotation and the Second Law of Thermodynamics," *Physical Optics*, **2nd ed. (NY, Macmillan, 1929), p. 500.**

Lord Rayleigh has called attention to a curious disposition of apparatus which permits the passage of light in one direction, while refusing absolutely to transmit it in the reverse direction. Two Nicol prisms are mounted with their directions of vibration at an angle of 45˚. Between them consider a medium in a magnetic field of strength sufficient to produce a rotation of the plane of polarization of 45˚. The polarized light passing through one Nicol will be rotated into such a plane that the second Nicol will stop it. If we reverse the direction of the light the polarized vibration will be brought into the plane of vibration of the second Nicol and be freely transmitted, since the magnetized medium rotates the plane of polarization in the same absolute direction (i.e., considered without regard to the direction of propagation) whichever way the light be traveling.

It is thus possible to construct an apparatus through which we can see without being seen, provided we limit ourselves to monochromatic light. It might be supposed, at first sight, that, by making use of this contrivance, an exception to the second law of thermodynamics might be realized, a radiating body throwing more energy into a second body than it received in exchange and thus elevating its temperature. Wien considered this case in his treatment of the theory of radiation presented at the International Congress of Physics, held in Paris (1900), and offered certain ways of escape out of the difficulty. Lord Rayleigh shows, however (Nature, LXIV), that, after all, no difficulty really exists.

Rayleigh, *On the magnetic rotation of light and the second law of thermodynamics*, **Nature (London), Vol. 64, p. 577 (Oct. 10, 1901).**

In a paper published sixteen years ago I drew attention to a peculiarity of the magnetic rotation of the plane of polarization arising from the circumstance that the rotation is in the same absolute direction whichever way the light may be traveling. "A consequence remarkable from the theoretical point of view is the possibility of an arrangement by which the otherwise general

optical law of reciprocity shall be violated. Consider, for example, a column of diamagnetic medium exposed to such a force that the rotation is 45°, and situated between two Nicols whose principal planes are inclined to one another at 45°. Under these circumstances, light passing one way is completely stopped by the second Nicol, but light passing the other way is completely transmitted. A source of light at one point A would thus be visible at a second point B, when a source at B would be invisible at A, a state of things at first sight inconsistent with the second law of thermodynamics." (Phil. Trans. 176, p. 343, 1885; Scientific Papers, vol. II, p. 360) [Footnote: That magnetic rotation may interfere with the law of reciprocity had already been suggested by Helmholtz.] It is here implied that the inconsistency is apparent only, but I did not discuss it further.

In his excellent report (*Les lois théoriques du rayonnement, Rapports présentés au Congrès International de Physique*, Paris, 1900, vol. II, p. 29), W. Wien, considering the same experimental combination of Nicols and magnetized dielectric, arrives at a contrary conclusion. It may be well to quote his statement of the case.

"The magnetic rotation of the plane of polarization constitutes an exceptional case worthy of remark, as we can imagine a situation which contradicts Carnot's principle if there does not exist some unknown compensating factor.

Namely, let's make the following suppositions. Two bodies of equal temperature are surrounded by an adiabatic envelope. The rays that they reciprocally exchange traverse two Nicol prisms. Between these prisms is located a nonabsorbing material in which there is a magnetic field that causes the plane of polarization of the light to be rotated through 45°. The radiation from body 1 passes through Nicol 1. We assume that the totally internally reflected ray is not absorbed but is returned back upon itself by suitable mirrors. Having oriented the principal plane of the second Nicol so that it is parallel to the plane of polarization of the rays transmitted by the magnetic rotator, all of the light passing through this rotator is transmitted by Nicol 2. In consequence, half of the light emitted by body 1 will be incident on body 2.

The rays emitted by body 2 are split into two equal parts by Nicol 2. One half, as above, is returned back upon itself to body 2 after reflections. The other half, after its plane of polarization has been rotated by 45° in the same direction as the rays emitted by body 1, hits Nicol 1. But since the principal plane of this Nicol is perpendicular to the plane of polarization of the light, no radiation is passed by it, and we can send all of the light back to body 2.

Body 2 therefore receives 3 times as much energy as body 1. That is, body 2 receives the whole of its own radiation plus half of that of body 1, while body 1 receives only half of its own radiation. One of the bodies thus heats up more and more at the expense of the other." [My translation from French.]

Wien then suggests certain ways of escape from this conclusion, but it appears to me that the difficulty itself depends upon an oversight. It is not possible to send back to 2 the whole of its radiation in the manner proposed. The second half, which after passing through Nicol 2 is totally reflected at Nicol 1 and then returned upon itself, on its arrival at Nicol 2 is not transmitted (as Wien seems to suppose) but is totally reflected. When again returned upon itself by a

perpendicular reflector and again rotated through 45° by the magnetized medium, it is in a condition to be completely transmitted by Nicol 1, and thus find its way to body 1 and not to body 2 as the argument requires. The two bodies receive altogther the same amount of radiation, and there is therefore no tendency to a change of temperature.

Although I have not not been able to find any note of it, I feel assured that the above reasoning was present to my mind when I wrote the passage already cited.

Afterword

When I first thought of this puzzle, I hoped to get around Rayleigh's solution by disposing of the totally reflected light from any given Nicol prism in some way other than returning it back upon itself, namely by providing a separate path either back to its source or dumping it outside of the reflecting envelope surrounding the sample. Unfortunately, closer thought has now convinced me that any such alternate path will necessarily open up new channels for irradiating the sample. For example, any bypass dumping light out of the cavity will let other light leak in.

Two last comments. Halliday, Resnick, and Krane, 4th ed., p. 958 says, "The principle of optical reversibility ... states that if there is no absorption of light, a light ray that is reflected or refracted will retrace its original path if its direction is reversed." Well, the ideal Faraday isolator I described at the beginning of this document involves no absorption and yet light cannot retrace its path through this one-way valve. I suggest correcting HRK by striking out "its direction" and replacing those words with "time" since reversing time reverses both the magnetic field and the direction of light propagation, which will then truly cause the light to retrace itself. But does this fix all possible scenarios in optics? Are there any instances of time-reversal asymmetry in optics? Are there any practical instances where I need to generalize further and consider CPT invariance? I am not aware of any, but there could be.

Finally, Rayleigh mentions Helmholtz reciprocity in his footnote. That's an interesting sideline in its own right. The technical statement of this theorem states that the bidirectional reflectance distribution function is symmetric in the incident and reflected angles. More simply, if a material scatters some fraction of the light incident from a narrowly defined solid-angle cone 1 into a second cone 2 (per projected solid angle of this outgoing cone), then that surface should reflect the same amount of light incident from cone 2 into cone 1. Unfortunately, it is clear that not all media exhibit Helmholtz reciprocity—to wit, our Faraday isolator! However, a controversy continues to rage in the literature as to whether or not that's just an oddball counter-example of little practical import. For a recent discussion of this issue, see W.C. Snyder, *Reciprocity of the bidirectional reflectance distribution function (BRDF) in measurements and models of structured surfaces*, IEEE Trans. Geosci. Remote Sensing vol. 36, p. 685 (1998).

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