

**Connecting the Work-Energy Theorem to the First Law of Thermodynamics**  
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Consider a system composed of  $i = 1, \dots, N$  particles. Integrating Newton's second law over space (in an inertial reference frame) gives the work-kinetic energy theorem for the  $i$ -th particle,

$$\int \vec{\mathbf{F}}_i \cdot d\vec{\mathbf{s}}_i = \Delta K_i$$

where  $\vec{\mathbf{F}}_i$  is the net force on particle  $i$ , and  $K_i \equiv \frac{1}{2} m_i v_i^2$  where  $m_i$  is the mass of particle  $i$  and  $\vec{\mathbf{v}}_i \equiv d\vec{\mathbf{s}}_i / dt$  is its velocity. Now sum this over all particles and split the net force into two terms,

$$\sum_{i=1}^N \int [\vec{\mathbf{F}}_{\text{ext}_i} + \vec{\mathbf{F}}_{\text{int}_i}] \cdot d\vec{\mathbf{s}}_i = \sum_{i=1}^N \Delta K_i$$

where "ext" and "int" refer to forces exerted by objects external to and internal to the system, respectively. But

$$\sum_{i=1}^N \int \vec{\mathbf{F}}_{\text{int}_i} \cdot d\vec{\mathbf{s}}_i = \sum_{i=1}^N \sum_{j=1}^N \int \vec{\mathbf{F}}_{\text{int}_{ij}} \cdot d\vec{\mathbf{s}}_i$$

where  $\vec{\mathbf{F}}_{\text{int}_{ij}}$  is the force that particle  $j$  exerts on particle  $i$  (with the prime indicating that we exclude the case  $i = j$ ). This is equal to

$$\sum_{i=1}^N \sum_{j=i+1}^N \int [\vec{\mathbf{F}}_{\text{int}_{ij}} \cdot d\vec{\mathbf{s}}_i + \vec{\mathbf{F}}_{\text{int}_{ji}} \cdot d\vec{\mathbf{s}}_j] = \sum_{i=1}^N \sum_{j=i+1}^N \int \vec{\mathbf{F}}_{\text{int}_{ij}} \cdot d(\vec{\mathbf{s}}_i - \vec{\mathbf{s}}_j) \equiv - \sum_{i=1}^N \sum_{j=i+1}^N \Delta U_{ij}$$

using Newton's third law in the second step, and assuming that all forces between particles are conservative<sup>1</sup> to get the third equality. We now define the external work as

$$W_{\text{ext}} \equiv \sum_{i=1}^N \int \vec{\mathbf{F}}_{\text{ext}_i} \cdot d\vec{\mathbf{s}}_i.$$

We thus obtain

$$\boxed{W_{\text{ext}} = \Delta E}$$

where the total energy of the system is the sum of the kinetic and potential energies of every particle composing it,<sup>2</sup>

$$E \equiv \sum_{i=1}^N \left[ K_i + \sum_{j=i+1}^N U_{ij} \right].$$

This is often rewritten in terms of macroscopic observables<sup>3</sup> as the first law of thermodynamics,

$$W_{\text{thermo}} + Q_{\text{thermo}} = \Delta E_{\text{thermal}} + \Delta E_{\text{nonthermal}}.$$

## Footnotes

1. For example, friction actually results from a sum over (conservative) electrostatic forces between the electron shells of the atoms in the contacting surfaces. As the name suggests, assuming that all of the internal forces between the particles are conservative is equivalent to assuming that energy is conserved. Otherwise it would be possible to deduce the law of conservation of energy from Newton's laws. Instead, we accept it as a postulated law consistent with all known experiments.

2. Full generality would require that we not only count the kinetic and potential energies (which includes the gravitational and electrostatic interactions) of the material particles, but also integrate over the nonconservative fields (such as the magnetostatic and radiation fields) and account for any nuclear transformations of particles.

3. How to divvy the external work between macroscopic work and heat, and how to partition the total energy into thermal and nonthermal portions is not at all trivial and is not handled the same way by all authors. For an example of the first, what fraction of the particle work done by kinetic friction is  $W_{\text{thermo}}$  and how much is  $Q_{\text{thermo}}$ ? For an example of the second, do the vibrations of a struck solid count as  $E_{\text{nonthermal}}$  or as  $E_{\text{thermal}}$ ? For these reasons, it is often better to use the boxed relation rather than the more conventional form of the first law.