

## Floating Cork in an Accelerating Elevator—C.E. Mungan, Fall 2014

A cork of cross-sectional area  $A$  and length  $H$  is floating in a beaker of water with its axis of symmetry oriented vertically. (Wine bottle corks do not normally float in that orientation, but you can force one to do so by driving short heavy nails into the bottom face to ballast it.) The beaker is placed in an elevator that is accelerating upward at say  $a = 0.3g$ . Does the cork float higher or lower in the water than when the elevator is stationary?

One might argue the cork floats lower as follows. The buoyant force on the cork is  $B = \rho_{\text{water}} Ahg$  where  $\rho_{\text{water}} = 1000 \text{ kg/m}^3$  is the density of water,  $g = 9.8 \text{ N/kg}$  is earth's surface gravitational field strength, and  $h$  is the submerged depth of the cork in the water. The gravitational force on the cork is  $W = \rho_{\text{cork}} AHg$  where  $\rho_{\text{cork}}$  is the density (or average density if ballasted) of the cork with  $\rho_{\text{cork}} < \rho_{\text{water}}$ . When the elevator is stationary, we must have  $B = W$  because the cork is floating in static equilibrium. Therefore the submerged fraction of the cork is

$$h_{\text{stationary}} / H = \rho_{\text{cork}} / \rho_{\text{water}} \equiv SG, \quad (1)$$

which defines the specific gravity of the cork. However, when the elevator is accelerating upward, we must instead have  $B - W = \rho_{\text{cork}} AHa$  which implies  $h_{\text{accelerating}} / H = 1.3SG$ . The submerged depth of the cork has increased 30%, which means the cork floats lower.

That answer is wrong. In fact, the cork continues to float at its original level, as can be seen in the YouTube video at <https://www.youtube.com/watch?v=OXuYWCWlaJI>. One way to rationalize the experimental result is to note that according to the equivalence principle, one would be unable to measure any difference between an elevator accelerating upward at  $0.3g$  on earth and another elevator at rest on the surface of a planet that has a surface gravitational field strength of  $1.3g$ . But the gravitational field strength canceled out of Eq. (1). Thus the cork would float at the same depth in an elevator at rest on earth as in an elevator at rest on this other planet.

Okay, but what is wrong with the argument made in the second paragraph above? Simply this: the buoyant force is no longer equal to  $\rho_{\text{water}} Ahg$  when the elevator is accelerating! To see why, note that the buoyant force originates from the hydrostatic pressure in the water. When the elevator is stationary, for example, we have  $B = pA$  where the gauge pressure (i.e., the pressure in excess of the atmospheric pressure pushing down on the top face of the cork) is  $p = \rho_{\text{water}} gh$ . Now recall the derivation of that hydrostatic pressure formula. One considers a cylinder of water that replaces the submerged portion of the cork. That is, its upper face is at the surface of the water and its lower face is at depth  $h$ . The gauge pressure on that lower face has to balance the weight of the cylinder of water divided by the cross-sectional area of the cylinder. Aha! That is no longer true when the elevator is accelerating upward. The gauge pressure must instead be

increased to  $p = \rho_{\text{water}}(g + a)h$  in order to accelerate the cylinder of water upward at  $a$ . Just as the equivalence principle predicted, we have to effectively increase  $g$  by  $a$  in the formula for  $B$ .

One last detail remains to make the whole argument satisfying. What is the physical mechanism that actually increases the hydrostatic pressure in the beaker of water? A bit of thought leads one to realize it is the increased normal force that the bottom of the beaker exerts on the water. In fact, although the formula  $p = \rho_{\text{water}}gh$  (for the unaccelerated case) suggests that gravity alone creates the pressure, one must really say that the normal force from the bottom is also critical. If I was holding the beaker in the air and its bottom suddenly broke off, the water would be in freefall and there would no longer be any pressure in it (ignoring surface tension keeping the water together). It might help to think instead of a stack of pennies in an otherwise empty beaker. There is only a “pressure” force pushing adjacent pennies together as long as the normal force from the bottom is acting. Drop the beaker and every penny will experience apparent weightlessness with zero “pressure” forces between them. To return to considering a beaker of water, by Newton’s third law the water gauge pressure at the bottom of the beaker multiplied by the cross-sectional area of the beaker equals the normal force that the bottom exerts upward to support the weight of the entire volume of water in the beaker. To accelerate that entire volume upward, we have to increase the normal force on the water and thereby increase the pressure in the water.

By the way, since our argument is based on the fact that the gravitational field cancels out of both  $B$  and  $W$ , will the cork’s depth of submersion remain unchanged even if the beaker were in freefall? If water were ideal, the answer would be yes. However, in reality, as already alluded to above, other effects such as surface tension and viscosity then dominate the the dynamics so that the cork sinks, as can be seen at <https://www.youtube.com/watch?v=cDAVOD9Vqz8> and explained in TPT **52**, 415 (2014).