

## Number of Galaxies in the Universe—C.E. Mungan, Summer 2011

Estimate the number of galaxies in the universe assuming that it is flat in the absence of dark matter and energy, that it is 13 billion years old, and that a typical galaxy contains 100 billion sun-like stars.

Imagine the universe to be a sphere centered on the earth whose surface has been expanding outward at the speed of light since the Big Bang, and is thus a distance of  $R = c\tau$  away, where  $\tau = 13 \times 10^9 \times 365 \times 24 \times 3600$  s. Hubble's law says that expansion speed is proportional to distance,  $v = Hr$  so that  $c = HR$ , and therefore the Hubble constant is  $H = 1/\tau$ . Thus the acceleration of the universe is described by  $a(r) \equiv dv/dt = Hv = H^2r$ . But if the universe is flat, then this acceleration is balanced by gravity,  $g(r) = Gm(r)/r^2$  where  $m(r) = \frac{4}{3}\pi r^3 \rho$  on length scales large enough that the universe can be treated as having a uniform mass density  $\rho$ .

Equating  $a$  and  $g$  implies that the (critical) density is

$$\rho = \frac{3H^2}{4\pi G} \quad (1)$$

which is a factor of 2 larger than the general relativistic expression. But the visible mass of the universe is  $M = \frac{4}{3}\pi R^3 \rho$ . Substituting in Eq. (1) and the preceding expressions for  $R$  and  $H$  leads to

$$M = \frac{c^3 \tau}{G}. \quad (2)$$

Dividing this result by  $10^{11}M_{\text{sun}}$  gives the final result for the number of galaxies,

$$\boxed{N = 8 \times 10^{11}}, \quad (3)$$

i.e., about 800 billion galaxies, which is comparable to the best current estimates, particularly if one reduces it by a factor of 2 to correct Eq. (1).