Inductance Calculations—C.E. Mungan, Spring 2003

As proven in AJP 67:320 (1999), the inductance of a device carrying total current I which can be subdivided into N filamentary loops carrying fractional currents $I_i = f_i I$ is given by

$$L = \sum_{i,j} f_i f_j l_{ij} \tag{1}$$

where the inductance of loop i due to loop j is denoted l_{ij}, which equals the self inductance L_i if $i = j$ and the mutual inductance M_{ij} if $i \neq j$.

For example, consider two inductors 1 and 2 in series, so that $f_1 = f_2 = 1$. The total inductance of the pair is then

$$L = (L_1 + M_{12}) + (L_2 + M_{21}) \tag{2}$$

as expected. Now apply this to a solenoid consisting of $i = 1,\ldots,N$ loops. The magnetic field due to the current I in loop j links loop i with flux $\int \vec{\mathbf{B}}_j \cdot d\vec{\mathbf{A}}_i$ and dividing this by I gives l_{ij}, so that

$$L = \sum_{i,j} \frac{\int \vec{\mathbf{B}}_j \cdot d\vec{\mathbf{A}}_i}{I} = \frac{\int \left(\sum_j \vec{\mathbf{B}}_j \right) \cdot \left(\sum_i d\vec{\mathbf{A}}_i \right)}{I} = \frac{\int \vec{\mathbf{B}} \cdot \left(N d\vec{\mathbf{A}} \right)}{I} = \frac{N}{I} \Phi \tag{3}$$

where $\vec{\mathbf{B}}$ is the total magnetic field inside the solenoid and $\Phi \equiv \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$ is the flux linking a single winding, giving the usual result.

As a second example, we can calculate the inductance of a hollow-core coaxial cable by chopping a cross section up into N thin wedges each carrying current fraction $f = 1/N$. Thus

$$L = \sum_{i,j} f^2 \frac{\int \vec{\mathbf{B}}_j \cdot d\vec{\mathbf{A}}_i}{I_i} = \frac{\int \left(\sum_j \vec{\mathbf{B}}_j \right) \cdot \left(\sum_i f \, d\vec{\mathbf{A}} \right)}{I} = \frac{\int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}}{I} = \frac{\Phi}{I} \tag{4}$$

where Φ is the flux linking a single wedge, again giving the usual result.

A rather less easy example is done in the AJP paper to demonstrate the power of this method, namely a coax cable where the inner conductor is a thick (rather than thin) shell. The basic idea is to divide it into a set of thin shells carrying fractional currents determined by the current density.