

Finding the Integral of an Inverse Function—C.E. Mungan, Summer 2016

I learned the following trick on Quora. Suppose you want to find the integral of $y(x) = f^{-1}(x)$ and already know the integral of the forward function $x(y) = f(y)$. In that case, you can simply use integration by parts,

$$\int y dx = xy - \int x dy. \quad (1)$$

Here is an example. We want to integrate $y = \tan^{-1} x$. Equation (1) gives us

$$\begin{aligned} \int \tan^{-1} x dx &= x \tan^{-1} x - \int \tan y dy = x \tan^{-1} x - \ln(\sec y) \\ &= x \tan^{-1} x - \ln \sqrt{1 + \tan^2 y} = \boxed{x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2)}. \end{aligned} \quad (2)$$

Nifty!