Suppose we wish to solve the following equation for \( x \),

\[
e^{ax} = bx + c
\]
given values of \( a, b, \) and \( c \). This equation can be rewritten as

\[
b(x + c/b)e^{-ax} = 1 \quad \Rightarrow \quad (-ax - ac/b)e^{-ax-ac/b} = -\frac{a}{b} e^{-ac/b}.
\]

So if we define two new variables

\[
z = -\frac{a}{b} e^{-ac/b} \quad \text{and} \quad W = -ax - ac/b
\]

then the equation becomes

\[
We^W = z
\]

whose inverse \( W(z) \) defines the Lambert \( W \) function. Thus the solution to Eq. (1) is

\[
x = -c/b - \frac{1}{a} \left( -\frac{a}{b} e^{-ac/b} \right).
\]

So to finish the problem, we need that function. One way to get it is graphically. Simply compute \( z(W) \) for a range from large negative to large positive values and plot it in Excel as \( W \) versus \( z \). That results in the following graph.
One sees that the function has two branches: the upper red one with domain $z \geq -1/e \approx -0.36788$ and range $W \geq -1$, and the lower blue one with domain $-1/e \leq z < 0$ and range $W \leq -1$.

For example, suppose one wishes to solve

$$e^x = 2x + 3$$

so that $a = 1$, $b = 2$, and $c = 3$. Then there are two solutions, given by

$$x = -1.5 - W\left(-0.5e^{1.5}\right) \approx -1.5 - W\left(-0.111565\right).$$

(7)

By trial and error in Excel one finds that

$$W\left(-0.111565\right) \approx \begin{cases} -3.42394 & \text{for blue branch} \\ -0.236625 & \text{for red branch} \end{cases}$$

(8)

so that

$$x \approx 1.92394 \text{ or } -1.373375.$$  

(9)

Another way to get the solution is using software such as WolframAlpha where the solutions are

$$x = -1.5 - \text{ProductLog}\left[-0.5e^{1.5}\right]$$

(10)

for the principal red branch, and

$$x = -1.5 - \text{ProductLog}\left[-1-0.5e^{1.5}\right]$$

(11)

for the fully negative blue branch.