Large Exponents in Scientific Notation—C.E. Mungan, Fall 2019

Suppose we want to write a large number written in terms of arbitrary bases instead in more standard stacked scientific notation as

$$a^{b^{c^{\cdot^{\cdot^{\cdot^{d}}}}}} = 10^{10^{10^{\cdot^{\cdot^{\cdot^{F}}}}}} . \tag{1}$$

Proceed in steps starting from the bottom up. First rewrite

$$a^b = 10^B \tag{2}$$

using logarithms. That is, take the log (base 10) of both sides of Eq. (2) to get

$$B = b \log a \tag{3}$$

so that Eq. (1) becomes

$$10^{B^{c^{\cdot^{\cdot^{\cdot^{d}}}}}} = 10^{10^{10^{\cdot^{\cdot^{\cdot^{F}}}}}} . \tag{4}$$

We can now repeat the process for

$$B^c = 10^C \tag{5}$$

and so on until we end at F.

For example, suppose we want to calculate the tunneling probability given approximately by

$$T = 16 \frac{E}{U_0} \left(1 - \frac{E}{U_0} \right) e^{-2L\sqrt{2m(U_0 - E)}/\hbar} . \tag{6}$$

Say the numbers work out to be

$$T = 5.164 e^{-259.4} . \tag{7}$$

An ordinary calculator will give zero if one tries to compute this number using the exponential function button. One alternative is to use software such as WolframAlpha which gives the final answer in Eq. (9) below. A better option so that one can use a calculator is to compute the natural logarithm of both sides of

$$e^{-259.4} = 10^{-n} \quad \Rightarrow \quad n = \frac{259.4}{\ln 10} = 112.656 \tag{8}$$

so that

$$T = (5.164)(10^{-0.656}) \times 10^{-112} = 1.140 \times 10^{-112} . \tag{9}$$