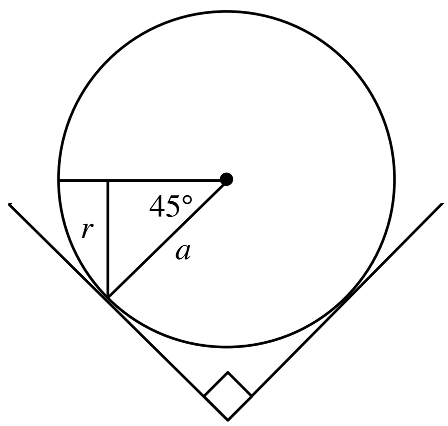
The Marble Loop-the-Loop on Angle Track—C.E. Mungan, Fall 2014

For an interrupted pendulum to smoothly execute a loop-the-loop of radius R, it must start at a height of $2.5R$ above the bottom point because the bob simply translates around in a circle. For a solid marble on a flat-bottomed track, it must start at $2.7R$, a little higher because it rolls and so needs extra energy for the rotations. However, a common way to keep the marble on the track is to use a 90-degree angle bracket as sketched below. Here I show the reduced effective radius of rotation of the ball means it must rotate faster and hence start yet higher at $2.9R$.



The radius of the solid ball is a. (Note: It helps to use a plastic ball rather than a metal one so that it grips the track better. As long as the ball rolls without slipping and the track does not shake, there will be no mechanical energy dissipation and the results here will be exact. Make sure however to use a solid ball not a hollow one!) The ball makes contact at two points that are a perpendicular distance $r = a/\sqrt{2}$ from the axis of rotation of the ball passing through its center of mass. Apply conservation of mechanical energy if the ball starts from rest at height h above the bottom point of the loop and just makes it around the top point of the loop that is $2R$ higher up,

$$mgh = \frac{1}{2}m\upsilon^2 + \frac{1}{2}\left(\frac{2}{5}ma^2\right)\left(\frac{\upsilon}{r}\right)^2 + mg(2R) \tag{1}$$

where m is the mass of the ball and υ is the speed of its center of mass (com) at the top of the loop. The quantity in the first parentheses is the moment of inertia of the ball about its com, and the term in the second parentheses is the angular speed of the ball at the top of the loop. Substituting in $r = a/\sqrt{2}$, Eq. (1) rearranges into

$$\upsilon^2 = \frac{10}{9}g(h-2R). \tag{2}$$

If the ball just makes it around the top of the loop, then it is starting to lose contact with the loop at that point, so that the normal force on the ball is zero. In that case, the centripetal force is provided by gravity alone, so that

$$mg = m\frac{\upsilon^2}{R}\ . \qquad (3)$$

Substituting Eq. (2) into the left-hand side and simplifying gives the desired result.