Newton's Laws—C.E. Mungan, Fall 2001

In this document I give an outline of a one-lecture presentation of Newton's laws (labeled N1 through N3) in an introductory mechanics course for physics majors.

Acceleration was already defined in the chapters on kinematics as the second time derivative of position measured with respect to some explicitly or implicitly stated coordinate system.

Force is a push or pull in some direction and is quantified as the reading on a normally oriented spring scale. The scale is calibrated in steps of some standard unit force.

We define an inertial reference frame[1] as one in which any object initially at rest remains at rest if no forces are applied to it.[2] (One may label this N1a if one wishes, but note that it is a definition not an experimental law.) Any attempt to define an inertial frame as one which is not accelerating is futile, because we have no absolute coordinate system against which the acceleration of the frame of interest can be measured, unless one wishes to invoke the nebulous concept of the "fixed stars."

Experimentally it is found that in an inertial frame, when various forces are successively applied to a given object, the resulting acceleration of the object is proportional to the force.[3] However, if we apply the same force to various objects, in general their accelerations are not the same. Some property of an object determines its response to an applied force and we call this property its inertial mass. We arbitrarily choose a specific lump of platinum-iridium to have one standard SI unit of inertial mass which we call 1 kg; a newton is then defined as that net force which causes a 1-kg mass to accelerate at 1 m/s^2. We can subsequently quantify the inertial mass of any other object by comparing the acceleration of the object of interest to that of the standard mass,

$$m_{object}(\text{kg}) = a_{standard} / a_{object} \,, \tag{1}$$

when the same net force is applied to both. Note that the object's mass is inversely proportional to its acceleration in Eq. (1) because inertia is a measure of an object's resistance to acceleration: a larger mass will have a smaller acceleration.

Now that we have definitions of force, mass, and acceleration, we are ready to state N2: "The acceleration of an object equals the net force applied to it divided by the inertial mass of the object, provided that the acceleration is measured in an inertial reference frame." All known sub-relativistic experiments are consistent with this axiom. Hence we glorify it with the name of "law" although we cannot derive or prove it in any absolute sense. For brevity, we refer to an object upon which the net force is zero as "isolated." This does not necessarily mean no forces act on it, but only that all forces are balanced so that they vectorially sum to zero.[4] Note that if an isolated object is at rest with respect to an inertial observer, then the second law implies that the acceleration of the object is always zero and the object must remain at rest. Hence N2 is consistent with our definition N1a. Such a situation is called "static translational equilibrium." Another special case is an isolated object moving with some nonzero velocity in an inertial frame. According to N2, its acceleration must again be zero and hence the object will move with constant velocity. This is called "dynamic translational equilibrium" and if one were so inclined one could label it N1b. It implies that if an arbitrary frame B is moving with constant velocity relative to some inertial frame A, then frame B is also an inertial system. This is an experimentally testable hypothesis which some people call Newton's first law, but I prefer to consider it merely a derived result from N1a and N2, rather than to call it an independent law.

Next consider in more detail how to compute the net force on some composite object. Suppose it is composed of elementary pieces A, B, ... Then the net force on the whole object is

$$\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \cdots = \left(\vec{F}_A^{ext} + \vec{F}_{AB} + \cdots\right) + \left(\vec{F}_B^{ext} + \vec{F}_{BA} + \cdots\right) + \cdots, \tag{2}$$

where the first doubly-subscripted force means, for example, the force on object A due to object B. But N3 states, "The force of any object A on any object B with which it is in contact is equal in magnitude and opposite in direction to the force of object B back on object A." In mathematical form this says

$$\vec{F}_{AB} = -\vec{F}_{BA}. \tag{3}$$

Inserting this into Eq. (2), we see that all internal forces will pairwise cancel so that

$$\vec{F}_{net} = \vec{F}_A^{ext} + \vec{F}_B^{ext} + \cdots \equiv \vec{F}_{net}^{ext} \tag{4}$$

which (fortunately for us) makes it practical to calculate the net force on composite objects.

Note the qualification in N3 that the two objects be in contact. If, for example, the sun were suddenly to be teleported across the galaxy, the earth would continue to merrily follow its usual orbit for a bit over 8 minutes, the time it takes information traveling at the speed of light (at which we assume gravity waves propagate) to get from the sun to the earth. That is, for this "action at a distance," the earth continues to feel the force of the sun for 8 minutes, but obviously the relocated sun is not feeling earth's force: N3 does not apply. Unfortunately ALL forces in modern physics act at a distance—at the microscopic level there is always space between atomic particles (which quantum mechanics tells us are not really corporeal in the classical sense in any case). To save N3 we therefore introduce the concept of fields. An object does not directly act at a distance on another object. Instead it sets up a field which propagates out to the other object and acts locally upon it. But fields are not merely a convenient fiction to resurrect N3; for example, as we will see next semester, they carry real energy and momentum in the form of electromagnetic waves.

[1] A reference frame can be thought of as a small train compartment, in which an observer sits and makes observations. The compartment must be small, so that it can be positioned at various locations in a nonuniform gravitational field or accelerating system. It can be represented by an infinite set of coordinate systems attached to the compartment, differing in choice of origin and rotation of the axes for example.

[2] Philosophically, it is not obvious how to tell that there is no force acting on an object. In practice, however, I assume we can work our way through a laundry list of possibilities—contact forces, gravitational forces, electromagnetic forces, etc. For further discussion of this issue, see L. Eisenbud, "On the classical laws of motion," AJP **26**:144 (1958).

[3] N2 can also be applied in noninertial reference frames, provided we introduce additional terms called "pseudoforces." Examples include the well-known centrifugal and Coriolis forces associated with rotating frames.

[4] The idea that we can linearly superpose individual forces as vectors to find the net force on an object, while intuitively appealing, cannot be proven theoretically but is subject to experimental verification. You might consider it another Newtonian law in fact.