

## Solving Newton's Second Law in One Variable in the Absence of Dissipation

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The goal of the problem considered here is to find the time required for a particle moving along a specified curve (possibly a straight line or a circle, but not necessarily) to move from one given point to another. This time will be denoted by  $T$  and might, for example, be the period of oscillation of a particle of mass  $m$  subject to a restoring force. However, the procedure can also be used for non-restoring forces, subject only to the restriction that purely conservative forces act on the particle so that mechanical energy is conserved.

Step 1: Draw a free-body diagram, unless the net force  $F$  is already clearly given in mathematical form. Write down  $F = ma$  along the direction in which the particle actually moves and specify how the force depends upon the position of the particle. If the particle's path is a straight line, this means expressing  $F$  as a function of  $x$  and writing  $a = \ddot{x}$ ; for a circular path, express  $F$  as a function of  $\theta$  and write  $a = \ddot{\theta}$ , specifying the relation between  $a$  and  $\alpha$ ; finally, for an arbitrary curved path of arclength  $s$ , we must write  $a = \ddot{s}$  and parametrize the curve, expressing both the force  $F$  and the element of arclength  $ds$  in terms of the parametrization variable (which could be, say,  $x$ ).

Step 2: Transform from this dynamics equation to the energy equation by using  $\ddot{\theta}d\theta = \dot{\theta}d\dot{\theta}$ , or an analogous equation for  $x$  or  $s$ , and integrating once. This is exactly the transformation you used in University Physics I to derive the work-energy theorem (cf. Halliday & Resnick Sec. 7.4). Do not forget the constant of integration—interpret this constant physically by considering the initial or boundary conditions (e.g., the initial speed of the particle, its maximum amplitude, etc.).

Step 3: Solve for  $\dot{\theta}$  (or the analogous speed variable) and integrate again by collecting all of the terms which depend on  $\theta$  on one side of the equation, obtaining  $T$  by relating it to  $\int dt$  which appears on the other side of the equation. In general, this  $\theta$  integral will not be elementary and you will have to express it as a special function and then approximate its value by making a power-series approximation or by looking its value up in a table, as appropriate for the particular problem at hand.