Coin Flips such that Heads are Nonconsecutive—C.E. Mungan, Summer 2026

A coin is flipped N times such that heads never occur consecutively. What is the probability that you get exactly n heads?

Both n and N must be non-negative integers with $n \le (N+1)/2$. For example, if the coin is flipped 3 or 4 times, you can get up to 2 heads.

If there are n heads then there must be $N-n$ tails. Arrange them in sequence as follows.

$$\mathrm{T}\ \ \mathrm{T}\ \ \mathrm{T}\ \cdots\ \mathrm{T}$$

There are $N-n+1$ positions before or after any of these tails. We have to put the n heads into those positions, which can be done in any of the following number of ways,

$$_{N-n+1}C_n = \frac{(N-n+1)!}{(N-2n+1)!n!} \tag{1}$$

which, for example, for $N=10$ and $n=4$ is equal to 35. The total number of ways to get any number of nonconsecutive heads for N flips of the coin is the $N+2$ Fibonacci number given by

$$\sum_{n=0}^{\mathrm{int}\frac{N+1}{2}} {}_{N-n+1}C_n = \frac{\left(1+\sqrt{5}\right)^{N+2} - \left(1-\sqrt{5}\right)^{N+2}}{2^{N+2}\sqrt{5}} \tag{2}$$

which, for example, for $N=10$ is equal to 144.

The answer is the ratio of Eq. (1) to (2). For example, the probability to get 4 heads in 10 flips is $35/144 = 24.3\%$.