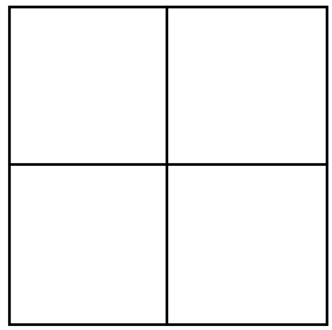
Number of Independent Kirchhoff Equations—C.E. Mungan, Spring 2016

The purpose of this document is to explain how to determine the number of independent Kirchhoff Current Junction (KCJ) and Kirchhoff Voltage Loop (KVL) equations for a particular circuit by inspection of it. For example, consider the following circuit, where each straight segment may contain batteries, resistors, capacitors, and inductors in series.

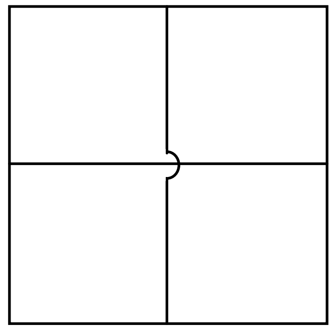


It consists of 5 junctions, 8 branches, and 13 loops (4 single squares, 4 adjoining double squares, 4 triple squares, plus the outside quadruple square) in all. A traditional problem might give all values of the battery emfs, resistances, capacitances, and inductances and ask for all currents, of which there is one per branch and hence 8 in all. In this case, one has 4 independent KCJ and 4 independent KVL equations, so that the number of equations and unknowns properly match.

The general rule is if J is the number of junctions then the number of independent KCJ equations is $J-1$ because we can always get KCJ at the last junction as the negative sum of the KCJ equations at the other junctions. Any current that runs "out of" one junction must run "into" an adjacent junction. Hence that current cancels out of a sum over both junctions, and we will be left only with the currents into and out of the last junction.

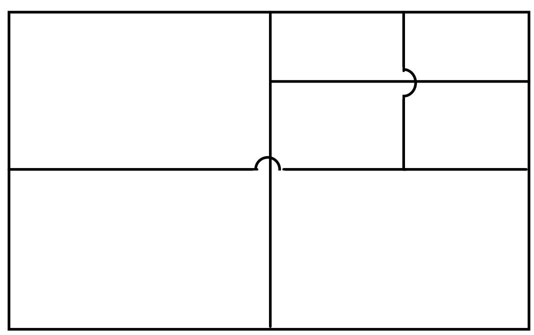
Counting the number of independent KVL equations is a bit trickier. If B is the number of branches, it has to be $B-J+1$. That guarantees the total number of independent Kirchhoff equations matches the number of unknowns, but it is not easy to directly justify that count from inspection of a circuit. A simpler way, that works provided no wire crosses over another wire, is to count the smallest closed loops that span the entire circuit. For example, in the preceding circuit, there are 4 small squares that "tile" it, and that is indeed the number of independent KVL equations.

However, what happens if there are "crossing" wires, such as the vertical one in the following circuit?

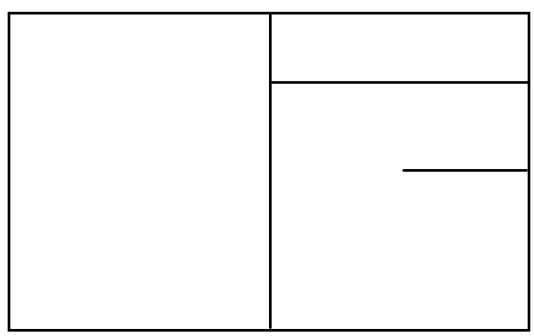


This circuit has 4 junctions, 5 loops, and 6 branches. Thus there should be 3 independent KVL equations. Here is one way to count those equations. Mentally delete all crossing wires (where a crossing wire is defined to run from the crossing to the nearest junctions in each direction). Now count the number of smallest tiling loops. Say you get N of them. Then return to the original circuit and this time delete all the wires that were crossed over. Again count the number of smallest tiling loops. Say you get M of them. Finally the number of independent KVL equations is $N+M-1$ (assuming that both N and M are nonzero). For example, in the preceding circuit, we have $N=2$ without the interior vertical wire, and we have $M=2$ without the interior horizontal wire, resulting in the correct answer of $N+M-1=3$.

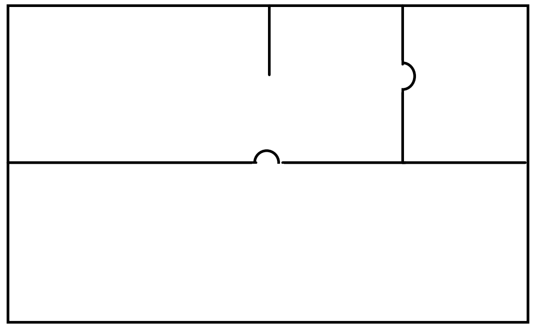
Here's a more complicated example circuit.



Deleting the two crossing wires shows that $N=3$ as in the next diagram.



Likewise deleting the two crossed over wires shows that $M=3$ from the following sketch.



Hence the original circuit has 5 independent KVL equations.

This procedure assumes no wire crosses over another crossing wire. We could find a rule for such “higher order” crossings, but it is probably not worth the trouble, as it seldom arises in practical circuits.