A method is here proposed for locating a position-sensitive detector (PSD), consisting of a two-dimensional \((X,Y)\) array, on a platform which can be rotated through known angles about a vertical axis. Denote the intersection between the axis of rotation and the plate as the origin \((x = 0, y = 0)\) of a coordinate grid on the surface of the plate. Suppose the PSD is aligned as described below, with a laser beam of negligible diameter striking it at a point that we designate as the center \((X = 0, Y = 0)\) of the array, perhaps by pushing a “zero” button on the detector amplifier. That center point is located at radial distance \(r\) and azimuthal angle \(\theta\) relative to the origin on the platform. The objective is to determine \(r\) and \(\theta\) by making measurements of the beam location \(X\) on the PSD as the plate is rotated through known angles \(\phi\) clockwise or counterclockwise relative to the initial position of the detector.

The PSD is initially aligned as follows. It is coarsely aligned so that the laser beam strikes near its center. The tip and tilt of the flat array are then adjusted until the detector back-reflects the laser toward its source (say to within \(1^\circ\) to avoid causing laser instabilities by exactly back-reflecting the beam into the laser cavity). These adjustments ensure the laser beam strikes the detector normally. Define the plate’s \(x\)-axis to be directed parallel to the back-reflected beam, and define its \(y\)-axis to point in the direction of positive \(\theta\). The setup is thus as sketched below.

Finally the rotation of the PSD about the beam axis is adjusted until the laser spot scans purely along the \(X\)-axis of the array as the plate is rotated. The detector alignment is now complete.
Now suppose we rotate the plate so that $\theta$ is increased to $\theta + \phi$. The PSD then rotates to a new position as shown in the following diagram. For clarity only the front (sensing) surface of the array is drawn.

![Diagram of plate rotation](image)

The beam spot moves to a new location $X$ on the array detector given as follows. The $y$-position of the center of the detector has increased by the distance

$$d = r \sin(\theta + \phi) - r \sin \theta.$$  

Since the detector is now inclined at angle $\phi$ relative to the $y$-axis, it follows that

$$X = \frac{d}{\cos \phi} = \frac{x \tan \phi - y}{\sec \phi - 1}$$

where $(x, y)$ are the initial coordinates of the center of the PSD, which consequently can be determined by making two measurements. However, a practical difficulty is that we are only going to be able to rotate the plate by a small angle before the beam sweeps off the end of the array, located at say $X_{\text{max}}$. Make one measurement of the value of $X$ when $\phi$ is chosen to give a beam position near $X_{\text{max}} / 10$ — call that data pair $(\phi_1, X_1)$. Next make a measurement of the value of $X$ when $\phi$ is chosen to give a beam position near $X_{\text{max}}$ — call that data pair $(\phi_2, X_2)$. Plot $X$ versus $\phi$ (with $X$ in cm and $\phi$ in rad) and do a fit to Eq. (2) to obtain $x$ and $y$.

As a numerical example, suppose the detector is actually located $r = 20$ cm from the plate’s origin at an angle of $\theta = \pi / 4$ and that the half-length of the array is $X_{\text{max}} = 1$ cm. Then according to Eq. (2), a real pair of data points is $(\phi_1 = 0.007, X_1 = 0.0987)$ and
(\phi_2 = 0.07, X_2 = 0.9569) where it is assumed the detector has a position resolution of 1 \mu m. These two points can be fit to Eq. (2) in Mathematica to obtain
\[
x \approx 14.15 \text{ cm} \quad \text{and} \quad y \approx 14.35 \text{ cm}.
\]

The exact values are \( x = y = 20 \text{ cm} / \sqrt{2} \approx 14.14 \text{ cm} \). Not surprisingly, the fitted \( y \) value is less accurate than the \( x \) value, as one can easily see by performing a small-angle expansion of Eq. (2); for example, if we expand to first power in \( \phi \), then \( X \approx x\phi \) and we cannot determine \( y \) at all. To improve the fitted \( y \) value, one either needs a longer array or higher precision in the measurement of \( X_2 \). (I am assuming \( \phi \) can be easily determined to sufficiently high precision.) In contrast, \( x \) is already well given by simply dividing \( X_1 \) by \( \phi_1 \) as you can verify for the above data, so that no fit is actually necessary to obtain it.