

The Problem of a Boy Born on a Tuesday—C.E. Mungan, Fall 2014

(i) Suppose that at least one of the children in a two-child family is a boy. What fraction of such families have two boys? (ii) Suppose that at least one of the children in a two-child family is a boy born on a Tuesday. What fraction of such families have two boys? (iii) Suppose that the older child in a two-child family is a boy. What fraction of such families have two boys?

The remarkable result is the answers are *different* for these three questions! A particularly clear way to derive the three answers has been provided by Mike Schiraldi online at <http://mikeschiraldi.blogspot.dk/2011/11/tuesday-boy-problem-in-under-300-words.html>. I reproduce his grid below.

	OMB	OTB	OWB	ORB	OFB	OSB	ONB	OMG	OTG	OWG	ORG	OFG	OSG	ONG
YMB	BB	GB												
YTB	BB	GB												
YWB	BB	GB												
YRB	BB	GB												
YFB	BB	GB												
YSB	BB	GB												
YNB	BB	GB												
YMG	BG	GG												
YTG	BG	GG												
YWG	BG	GG												
YRG	BG	GG												
YFG	BG	GG												
YSG	BG	GG												
YNG	BG	GG												

The entries along the top row refer to the older (O) child being either a boy (B) or girl (G) born on Monday (M), Tuesday (T), Wednesday (W), Thursday (R), Friday (F), Saturday (S), or Sunday (N) for a total of 14 possibilities. Likewise there are 14 entries along the leftmost column for the younger (Y) child. The entries within each cell indicate whether we have a family with an older boy and a younger boy (BB), an older boy and a younger girl (BG), an older girl and a younger boy (GB), or an older girl and a younger girl (GG).

One can now read the answers off this grid. For question (i), ignore the blue highlighting. The grid divides into four equal quadrants separated by the bold lines: in three of these quadrants (BB, GB, and BG) we have families with at least one boy. Out of those three quadrants, only one (BB) has two boys. Thus the answer to the first question is $1/3$.

For question (ii), consider the cells highlighted in blue. Only those 27 cells correspond to families with at least one child being a boy born on a Tuesday. (Note carefully there are *not* 28 such cells, because OTB/YTB only occurs once and should not be double-counted!) Of those, only the 13 cells that are in the top left quadrant list families where both children are boys. So the answer to the second question is $13/27$, which is larger than $1/3$.

Finally for question (iii), we again ignore the blue highlighting and consider only the left two quadrants for which the older child is a boy. Of those, only the upper quadrant corresponds to two-boy families, so the answer to the third question is $1/2$, which is larger than $13/27$.

It would appear that additional knowledge changes the odds that a family has two boys. However, that is not always true! Consider one last question: (iv) Suppose that the older child in a two-child family is a boy born on a Tuesday. What fraction of such families have two boys? Now we consider only the vertical blue column in the preceding grid and we see right away that the answer to this question is $1/2$, the same as the answer to question (iii). This time, the additional knowledge about the Tuesday birthday did not change the probability.