

Quant Quiz—C.E. Mungan, Spring 2015

ref: <http://blogs.wsj.com/moneybeat/2015/03/02/do-you-have-what-it-takes-to-be-a-master-of-math/>

1. Consider 6 glasses standing in a row. The first 3 are filled with water, and the last 3 are empty. What is the minimum number of glasses that must be moved to change the row into alternating empty and full glasses?

- (a) 1 (b) 2 (c) 3 (d) 4

The out-of-box solution is (a): Pick up glass 2, pour its contents into glass 5, and then replace glass 2. Only glass 2 was thus moved. However, the straight math solution is (b): Swap glasses 2 and 5. If the point of this quiz is to test creativity, I am okay with solution (a). However, if the point is to test math, I think (b) is the right answer.

The second question only tests knowledge of movies, and specifically one which most tech geeks have probably not seen. So I have taken the liberty of rephrasing the question:

2. What is

$$\lim_{x \rightarrow 0} \frac{\ln(1-x) - \sin x}{1 - \cos^2 x}$$

as in the movie *Mean Girls*?

- (a) -1 (b) 0 (c) ∞ (d) it does not exist

Noting that the denominator is $\sin^2 x$, we can use the two standard Taylor series for small x to approximate $\ln(1+x) \approx x$ and $\sin x \approx x$ to rewrite the expression as

$$\lim_{x \rightarrow 0} \frac{-x - x}{x^2} = -2 \lim_{x \rightarrow 0} \frac{1}{x}$$

which is multivalued. It equals $+\infty$ approaching from $x < 0$, but $-\infty$ from $x > 0$. Hence the solution is (d) not (c). Alternatively, you can apply l'Hôpital's rule once to convert the given indefinite $0/0$ form, but that approach is slower and more prone to algebraic errors.

3. If you can trade three bottle caps of Super Fizz Soda for one fresh bottle of Super Fizz Soda, how many bottles of Super Fizz Soda can you drink starting with 27 caps?

- (a) 7 (b) 9 (c) 11 (d) 13

The initial 27 caps give you 9 bottles. The caps from them give you 3 more bottles. Finally their caps give you 1 additional bottle, for a total of 13.

4. What is the largest whole number n such that $7^{2048} - 1$ is divisible by 4^n ?

- (a) 1 (b) 3 (c) 5 (d) 7

This question is the hardest on the quiz. However, the fact that it is multiple choice enables us to deduce that the solution must be (d) as follows. Note that we are trying to simplify a repeated difference of squares,

$$(7^{1024} + 1)(7^{1024} - 1) = (7^{1024} + 1)(7^{512} + 1)(7^{256} + 1)(7^{128} + 1) \times \\ (7^{64} + 1)(7^{32} + 1)(7^{16} + 1)(7^8 + 1)(7^4 + 1)(7^2 + 1)(7 + 1)(7 - 1).$$

The final expression is a product of 12 terms, each of which is even, so we have twelve 2's or six 4's. Thus the answer must be (d). Note that the last 3 terms are 50, 8, and 6. The 8 has an additional factor of 4 in it. (The first 9 terms contain integer powers of $49^2 = 2401$ and hence will end with the digits 02 after unity is added to them. When divided by 2, we see that each term will thus be a number ending in 01 which is odd.)

5. For all real numbers x and y , the function f satisfies

$$f(x) + f(2x + y) + 5xy = f(3x - y) + 2x^2 + 1.$$

What is the absolute value of $f(-4)$?

- (a) 0 (b) 1 (c) 3 (d) 7

The trick is to eliminate one of the f terms by choosing $y = 2x$ to get

$$f(4x) + 8x^2 = 1.$$

Substituting $x = -1$ now gives $|f(-4)| = 7$.