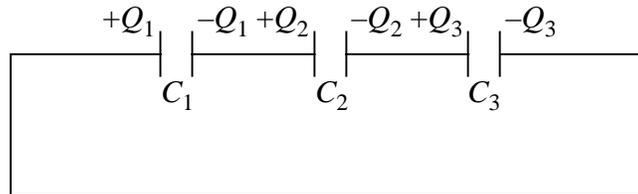


Solution to Tipler P25.59—C.E. Mungan, Spring 2002

Three capacitors ($C_1 = 2 \mu\text{F}$, $C_2 = 4 \mu\text{F}$, and $C_3 = 6 \mu\text{F}$) are each separately charged up to 200 V. They are then wired in series and the two end terminals of the set are shorted together, as sketched below. Find the equilibrium charge on and voltage across each capacitor.



The initial charges on each capacitor are found from the definition of capacitance,

$$Q_1 = C_1 \times 200 \text{ V} = 400 \mu\text{C}; \quad Q_2 = C_2 \times 200 \text{ V} = 800 \mu\text{C}; \quad Q_3 = C_3 \times 200 \text{ V} = 1200 \mu\text{C}. \quad (1)$$

Denote the final charges and voltages (which we are asked to find) by primes,

$$Q'_1 = C_1 V'_1; \quad Q'_2 = C_2 V'_2; \quad Q'_3 = C_3 V'_3. \quad (2)$$

To solve for these, we can apply our two rules about circuit elements in series. As we shall see, these are equivalent to Kirchhoff's rules:

1. ADD VOLTAGES.

The total potential drop across the set of elements equals the sum of the potential drops across each element. This is a statement of conservation of energy, because by definition the potential is the potential energy of a unit charge.

2. EQUAL CHARGE FLOW.

The charge which flows from one element to the next is the same for every element. This is a statement of conservation of charge, because charge cannot spontaneously appear or disappear in the wires or circuit elements and because minimization of the electric field energy requires that the two plates of a capacitor bear equal and opposite charges.

Since the left plate of capacitor 1 is connected (by an ideal wire) to the right plate of capacitor 3, the net voltage across the set of capacitors is zero. Hence, according to rule 1 we have

$$V'_1 + V'_2 + V'_3 = 0. \quad (3)$$

On the other hand, rule 2 says that the same amount of charge ΔQ flows through every connecting wire in the circuit, say in the counter-clockwise direction. Hence we see that

$$Q'_1 = Q_1 - \Delta Q; \quad Q'_2 = Q_2 - \Delta Q; \quad Q'_3 = Q_3 - \Delta Q. \quad (4)$$

Note that the second rule does NOT say that the final CHARGES are the same for the three capacitors. It says that the CURRENTS are equal. That is, every capacitor changes in charge by the same amount. Equal final charges only occur for initially uncharged capacitors in series—for that special case $Q_1 = Q_2 = Q_3 = 0$ initially, and hence Eq. (4) implies that the final charge on every capacitor must be the same.

The problem is now essentially done. Substitute each Eq. (4) into each Eq. (2) and divide each by its capacitance to get expressions for the final voltages in terms of ΔQ and knowns. Substitute these into Eq. (3) to get

$$\frac{Q_1 - \Delta Q}{C_1} + \frac{Q_2 - \Delta Q}{C_2} + \frac{Q_3 - \Delta Q}{C_3} = 0. \quad (5)$$

Solve this for ΔQ ,

$$\frac{\Delta Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} = 600 \text{ V}, \quad (6)$$

since the left-hand side is just the sum of the three initial voltages, namely 200 V each. Here I have introduced the equivalent series capacitance,

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} = \frac{12}{11} \mu\text{F}. \quad (7)$$

Hence one finds the final answers (which you should check),

$$\begin{aligned} Q'_1 &= -255 \mu\text{C}; & Q'_2 &= +145 \mu\text{C}; & Q'_3 &= +545 \mu\text{C} \\ V'_1 &= -127 \text{ V}; & V'_2 &= +36 \text{ V}; & V'_3 &= +91 \text{ V}. \end{aligned} \quad (8)$$

Note that the negative values for capacitor 1 mean that its polarity has reversed and should not be thought of as a violation of our convention in this chapter that charges and voltages are always taken to be positive for capacitors!