Rolling On and Off a Moving Surface—C.E. Mungan, Summer 2026

A cylindrically symmetric object is rolling without slipping on a stationary horizontal table. It rolls onto a horizontal surface that is moving (not necessarily at constant velocity) relative to the table. Later it leaves the surface and returns to the table. Eventually it resumes rolling without slipping on the table. Neglecting both air drag and rolling friction, prove that the final velocity (of the center of mass of the object abbreviated "com") is the same as its initial velocity.

Problems of this sort have been previously discussed as follows, in approximately chronological order of publication:

(i) A French Exploratium museum hosted by ANAIS built a rotating turntable which a rolling ball can cross, as shown in a video [1]. The ball exits the turntable with the same speed and direction of motion as the ball entered it. In fact, it has been claimed [2] that the exit path is simply a continuation of the entry path across the turntable (as would happen if the table were not turning) but in the video one sees that that is only approximately true. A later analysis [3] shows there is indeed a small lateral displacement of the path due to slipping as the ball moves on and off the turntable until it resumes rolling without slipping in both cases.

(ii) Ferguson [4] considers a basketball initially lying at rest on a sheet of newspaper on a table. If the sheet is pulled out from under the ball, the basketball quickly comes to a dead stop on the table (after a short phase during which the ball rolls *with* slipping) thereby matching its initial situation.[1] His analysis is notable for being brief and yet informative.

(iii) Singh [5] has an instructive discussion of a related problem in her two appendices. She shows that three different methods give complementary insights: conservation of angular momentum for an axis passing through the floor; force and torque due to friction combined with the equations of translational and rotational kinematics; and a work-energy approach. Although she initially considers only a hoop rotating without translating on a table, in the end she generalizes both to an arbitrary shape factor γ (defined below) and to an object which is initially translating and rotating.

(iv) Morin [6] provides a proof of the assertion in the first paragraph above for a uniform ball, as summarized below more generally.

(v) Tokieda [7] like Ferguson discusses a cylinder or beachball placed at rest on a surface which is subsequently yanked out from under it, either briskly or gently. Both of these authors consider the angular momentum about an axis attached to the floor, rather than about the com. For an axis in the y direction shown in Fig. 1 along the surface of the floor, friction produces zero torque on the object when the object is on top of a thin sheet of paper. Unfortunately, this reasoning leads Tokieda to *incorrectly* imply that the answer could be different if the surface were instead the top of a thick book placed on the table (because the frictional torque would then be nonzero). In fact, no significant difference is seen for the final velocity of a pen placed on a thin sheet and on a thick book in videos [8]. Both the horizontal linear and angular velocity of the pen cannot change while it is in the air! If the pen starts on a book, its subsequent impact with the table cancels the vertical component of the velocity and thus the angular momentum the pen gained while on the surface.

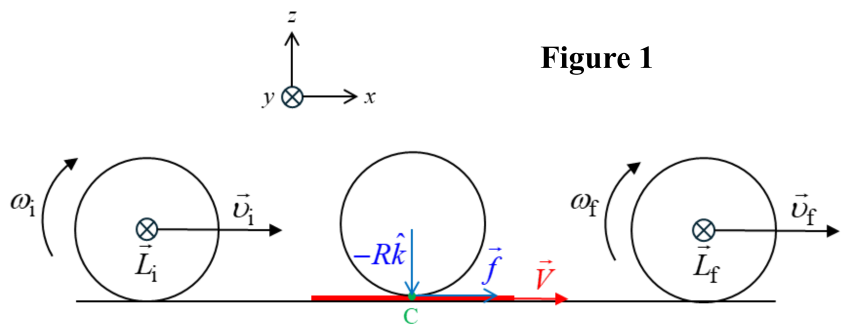
---

[1] When I tried this experiment, the ball did not tend to come to a full stop, but that could be because either I did not keep the paper fully horizontal or the table was not sufficiently smooth and level.

(vi) Cross [9] again considers a uniform ball placed at rest on a sheet of paper on a table. The sheet is then pulled rightward. He analyzes and measures the subsequent motion of the ball for two relevant[2] cases: the paper is uniformly accelerated such that the ball rolls without slipping on it (as summarized in the Appendix), and otherwise the ball rolls *with* slipping on the sheet. Also see De Luca [10] for a cylindrical can rolling without slipping on a conveyor belt.

(vii) Finally, a recent article [11] proposes an experiment that I independently thought of trying. A cylinder is placed on top of a rolling cart on a horizontal track that is pulled by a string connected around a pulley to a hanging weight. The constant accelerations of both the cart and cylinder are measured and compare favorably to the theory presented in the Appendix.

Figure 1



To begin the analysis, we are given that air drag and rolling friction are negligible, and the table and surface are horizontal. Consequently, the only horizontal force that acts on the object is either kinetic or static friction. Kinetic friction occurs if and only if the object is slipping relative to the table or surface. However, static friction can *never* occur on the table for the following reason. If static friction pointed forward, it would simultaneously translationally accelerate and angularly decelerate the object, and vice versa if static friction pointed backward; in either case, the contact point between the object and the table would then not remain at rest and so the friction could not be static! (In contrast, as shown in the Appendix, static friction *can* occur when the object is on the surface, but only if the surface is accelerating.) In any event, designate time t_i as the instant at which the object rolls onto the surface, and time t_f as the earliest instant after the object has left the surface that it is once again rolling without slipping on the table. In other words, there can only be friction on the object at times between t_i and t_f. At any such intermediate time t the friction can be zero, static, or kinetic, and so it will be designated as $\vec{f}(t)$ in general to cover all possibilities.

With that prelude, the essence of Morin's solution summarized by Seth Rittenhouse, is as follows. The rolling object has mass m, radius R, and moment of inertia (about its com) $I = \gamma m R^2$. For example, $\gamma = 0.4$ for a solid sphere or $\gamma = 0.5$ for a solid cylinder. Its initial linear momentum is $\vec{p}_i = m\vec{\upsilon}_i$ where $\vec{\upsilon}_i$ is the translational velocity of its com. Its initial angular momentum (about its com) has magnitude $L_i = I\omega_i = \gamma m R \upsilon_i$ where its initial angular speed is $\omega_i = \upsilon_i / R$ because it is rolling without slipping on the table. In vector form, by referring to the coordinate system in Fig. 1, this initial angular momentum can be written as

[2]Cross mentions another case where the paper is pulled slowly enough that the ball translates rightward with the sheet without rotating. However, that requires the average contact force (which is the sum of the normal and frictional forces) to act at a point around the circumference of the ball to the left of its lowermost point, such that the force is directed forward while passing through the com of the ball. As he points out, that would arise from an indentation of the paper behind the ball and is thus not relevant under the present assumption that the surface is strictly horizontal.

$$\vec{L}_{\rm i} = \gamma R\hat{k} \times \vec{p}_{\rm i} \,. \tag{1}$$

Likewise, its final linear momentum is $\vec{p}_{\rm f} = m\vec{\upsilon}_{\rm f}$ and its final angular momentum is

$$\vec{L}_{\rm f} = \gamma R\hat{k} \times \vec{p}_{\rm f} \tag{2}$$

after it resumes rolling without slipping on the table, now at angular speed $\omega_{\rm f} = \upsilon_{\rm f} / R$. Therefore its change in angular momentum is

$$\Delta\vec{L} = \gamma R\hat{k} \times \Delta\vec{p} \,. \tag{3}$$

It is the (time-varying) frictional force $\vec{f}$ that produces this change via the torque $\vec{\tau}$ it creates about the com,

$$\Delta\vec{L} = \int_{t_{\rm i}}^{t_{\rm f}} \vec{\tau}\, dt = -R\hat{k} \times \int_{t_{\rm i}}^{t_{\rm f}} \vec{f}\, dt = -R\hat{k} \times \Delta\vec{p} \tag{4}$$

using the impulse-momentum theorems for both translations and rotations. Equating the right-hand sides of Eqs. (3) and (4) implies

$$(1+\gamma)R\hat{k} \times \Delta\vec{p} = 0 \,. \tag{5}$$

Since $(1+\gamma)R\hat{k} \neq 0$ and $\hat{k} \perp \Delta\vec{p}$, the only way this expression can be zero is if $\Delta\vec{p} = 0 \;\Rightarrow\; \vec{\upsilon}_{\rm f} = \vec{\upsilon}_{\rm i}$ as we wanted to show.

Morin's solution can be greatly simplified, as shown in several of the journal articles referenced above, by considering the angular momentum of the object about an axis attached to the surface of the floor rather than one attached to the com. In that case, $\vec{f}(t)$ produces zero torque and so the angular momentum of the object never changes, whether it is on the floor or on the arbitrarily accelerated surface! Although there is both spin and orbital angular momentum of the object about an axis along the floor (instead of just spin angular momentum about an axis through the com), there is a fixed relationship between $\vec{\upsilon}$ and $\vec{\omega}$ for rolling without slipping, and thus $\vec{L}$ independently depends only on $\vec{\upsilon}$. Conservation of angular momentum then immediately implies that $\vec{\upsilon}_{\rm f} = \vec{\upsilon}_{\rm i}$.

Of course, while the object is on the surface, $\vec{\upsilon}$ will not in general be equal to $\vec{\upsilon}_{\rm i}$ but can instead vary in some complicated way. In particular, suppose the surface in the middle panel of Fig. 1 is moving at some nonzero velocity $\vec{V}$ at the instant the object rolls onto it. Then the translational velocity of the object *relative to the surface* given by $\vec{\upsilon}_{\rm i} - \vec{V}$ no longer matches the no-slip value $-R\hat{k} \times \vec{\omega}_{\rm i}$. Consequently the object must slip for at least a little while (and possibly for the entire time it is on the surface, depending on if and how $\vec{V}$ subsequently changes with time). Likewise, there must be slipping when the object leaves the surface, assuming the object is rolling without slipping on the surface which is moving relative to the table at that instant.

As a specific example of how to analyze one of these slipping phases of the motion, suppose that an object is rolling without slipping in the left-hand panel of Fig. 1 and it transitions onto the surface shown in the middle panel that is moving rightward at a constant speed of V. The contact point C has zero instantaneous speed relative to the table at the instant it rolls onto the surface and so it will slip backward relative to the forward-moving surface. Consequently there will be a forward kinetic frictional force of $f = \mu mg$ acting on the object at point C. That force will translationally accelerate the com of the object to some new speed

$$\upsilon = \upsilon_{\mathrm{i}} + \mu g t \tag{6}$$

in a time *t*, and the frictional torque will rotationally decelerate the angular speed of the object to

$$\omega = \omega_{\mathrm{i}} - \frac{\mu m g R}{\gamma m R^2} t . \tag{7}$$

The object will stop slipping on the surface when the translational speed of the object relative to the surface satisfies the no-slip condition,

$$\upsilon - V = \omega R \quad \Rightarrow \quad t_{\mathrm{slip}} = \frac{\gamma V}{(1+\gamma)\mu g} \tag{8}$$

using Eqs. (6) and (7), as well as the initial no-slip constraint $\upsilon_{\mathrm{i}} = \omega_{\mathrm{i}} R$ when the object is rolling on the table. As a check, this result correctly predicts there will be no slipping if the surface is not moving. Interestingly, the direction of $\vec{V}$ does not matter; only its speed V does. Tokieda uses Eq. (8) to estimate that a ball that rolls onto the ANAIS rotating platform will slip for about 0.1 seconds until it starts rolling without slipping again.

Appendix

A rigid and rotationally symmetric object of shape distribution factor γ (as defined in the main body of this paper) is rolling without slipping on the rigidly flat floor of a train accelerating at $\vec{a}_{\mathrm{train}}$ along a horizontal track. If the floor exerts a static frictional force on the rolling object, what must be the acceleration $\vec{a}_{\mathrm{com}}$ of the object's center neglecting air drag?

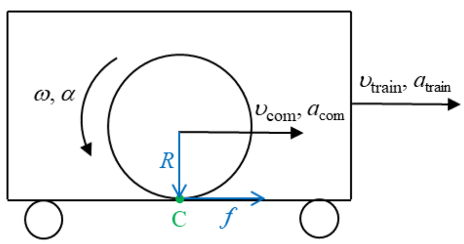


Figure A1

As Fig. A1 shows, Newton's second law for translations becomes

$$\vec{f} = m\vec{a}_{\mathrm{com}} \tag{A1}$$

relative to the inertial reference frame of the track, whereas Newton's second law for rotations implies that the magnitude of the object's angular acceleration is given by

$$Rf = \gamma m R^2 \alpha \quad \Rightarrow \quad \alpha = \frac{a_{\mathrm{com}}}{\gamma R} \tag{A2}$$

using Eq. (A1) in the second step. The contact point C on the object (where it touches the floor of the compartment) moves at speed

$$\upsilon_{\mathrm{C}} = \upsilon_{\mathrm{com}} + R\omega \tag{A3}$$

again relative to the track. If that contact point is instantaneously at rest relative to the train (so that the friction is static) then υ_{C} must be equal to $\upsilon_{\mathrm{train}}$, so that Eq. (A3) becomes

$$v_{\text{train}} = v_{\text{com}} + R\omega \tag{A4}$$

whose time derivative is

$$a_{\text{train}} = a_{\text{com}} + R\alpha = a_{\text{com}} + \frac{a_{\text{com}}}{\gamma} \tag{A5}$$

using Eq. (A2) in the last step. Thus the requested solution to the problem that began this Appendix is

$$a_{\text{com}} = \frac{\gamma}{1+\gamma} a_{\text{train}} \,. \tag{A6}$$

For example, for a solid ball, Eq. (A6) becomes $a_{\text{com}} = \frac{2}{7} a_{\text{train}}$. Substituting that back into Eq. (A1) implies that the frictional force has magnitude $f = \frac{2}{7} m a_{\text{train}}$, which is static provided $f \le \mu_s mg \;\Rightarrow\; a_{\text{train}} \le 3.5 \mu_s g$ where μ_s is the coefficient of static friction between the ball and floor. As a check, this equation for the static frictional force correctly implies that $f = 0$ for a ball rolling without slipping on a nonaccelerating horizontal surface (such as a tabletop). In the absence of air drag or rolling resistance, the ball would then roll at constant velocity forever [12].

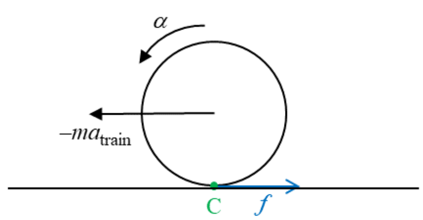


Figure A2

The "magic" that permits an accelerating surface to exert static friction on a rolling object becomes apparent when the left-hand panel of Fig. 1 is compared to Fig. A1. While in both cases the objects have a rightward translational velocity (relative to the earth), the directions of their angular velocities are opposite to each other! How can the ball aboard the train rotate the "wrong" way? The answer is seen by jumping into the reference frame of the train, as sketched in Fig. A2. In that frame, there is a pseudoforce of $-ma_{\text{train}}$ acting backward on the ball due to the forward acceleration of the train. Combining that with the forward frictional force of $+\frac{2}{7} m a_{\text{train}}$ for the specific example of a solid ball, the net force on the ball is $-\frac{5}{7} m a_{\text{train}}$ which is backward. Consequently the com of the ball translates backward *relative to the train* consistent with the fact that it rotates *counter-clockwise*. It is only *relative to the track* that the com of the ball translates forward at $a_{\text{com}} = \frac{2}{7} a_{\text{train}}$; subtracting from it the forward acceleration a_{train} of the train, the ball therefore translates backward at $-\frac{5}{7} a_{\text{train}}$ relative to the train. Ferguson extends this idea by noting that the magnitude of the acceleration of the com relative to the track divided by the magnitude of the acceleration of the com relative to the train is 2/5, or more generally γ. Thus if the object starts out at rest on the floor of a train at rest and if the train then accelerates forward with $a_{\text{train}} \le 3.5 \mu_s g$ such that it rolls without slipping, the distance the object will travel relative to the track is proportional to γ. So if a hollow hoop and a solid disk are started side by side at rest on a stationary page and the page is then pulled rightward until both roll without slipping off the left edge of the page and end up at rest on the table, the hoop will have traveled rightward twice as far as the disk compared to their common starting point relative to the table! That makes for a nice contrast with rolling them down a ramp without slipping starting from rest. The distance traveled along the incline in that race is proportional to $1/(1+\gamma)$ for a given time interval, and so the disk will outrace the hoop. By the time the disk is at the bottom of the incline, the hoop will only be three-quarters of the way down the ramp.

As a check, suppose the train starts out at rest and has a constant forward acceleration of $a_{\text{train}} < (\gamma+1)\mu_s g/\gamma$. At any later instant in time when the train is moving at speed υ_{train}, the object rolling without slipping on its floor has orbital angular momentum (about an axis passing through the floor of the train but fixed relative to the track) of $mR\upsilon_{\text{com}}$ directed into the page and spin angular momentum of $\gamma mR^2\omega = mR\upsilon_{\text{com}}$ directed out of the page for a total angular momentum of zero, the same as it started with when the train was at rest. In this case, let's find the time T that it takes the object's com to roll backward a distance d relative to the train. The magnitude of the acceleration of the com of the object relative to the train is

$$a_{\text{rel}} = a_{\text{train}} - a_{\text{com}} = \frac{a_{\text{train}}}{1+\gamma} \tag{A7}$$

using Eq. (A6) where the order of subtraction was reversed to make a_{rel} positive. Since the object starts from rest, the time is thus

$$d = \frac{1}{2}a_{\text{rel}}T^2 \quad \Rightarrow \quad T = \sqrt{\frac{(1+\gamma)2d}{a_{\text{train}}}}\,. \tag{A8}$$

During this same time, the distance the train moves along the track is

$$x = \frac{1}{2}a_{\text{train}}T^2 = (1+\gamma)d \tag{A9}$$

so that the distance the object moves forward relative to the track is $D = x - d = \gamma d$, as Ferguson has shown in a much simpler way.

[1] https://www.youtube.com/watch?v=IVKhu7dInJQ

[2] J.-M. Lévy-Leblond, "Le billard d'ANAIS," Eur. J. Phys. **7**, 252–258 (1986).

[3] A.P. Ivanov, "The ANAIS billiard experiment," Dokl. Phys. **61**, 285–287 (2016).

[4] J.L. Ferguson, "Pulling the rug from under round objects," Phys. Teach. **39**, 224–225 (2001).

[5] C. Singh, "When physical intuition fails," Am. J. Phys. **70**, 1103–1109 (2002).

[6] D. Morin, *Introduction to Classical Mechanics* (Cambridge University Press, 2008) problem 9.29 on page 421 with its solution on pages 453–454.

[7] T. Tokieda, "Roll models," Amer. Math. Monthly **120**, 265–282 (2013).

[8] https://physics.stackexchange.com/questions/444063/which-way-does-a-cylinder-on-a-slab-roll-after-the-slab-is-removed

[9] R. Cross, "Motion of a ball on a moving surface," Phys. Teach. **54**, 76–79 (2016).

[10] R. De Luca, "Physics at the grocery store," Rev. Bras. Ensino Fis. **44**, e20220086 (2022).

[11] M. Holovko, S. Kryzhanovskyi, and V. Matsyuk, "The study of the motion of bodies in a noninertial reference frame," Phys. Educ. **61,** 035008 (2026).

[12] D.C. Giancoli, *Physics for Scientists & Engineers*, 4th ed. (Pearson, 2009) section 10.10.