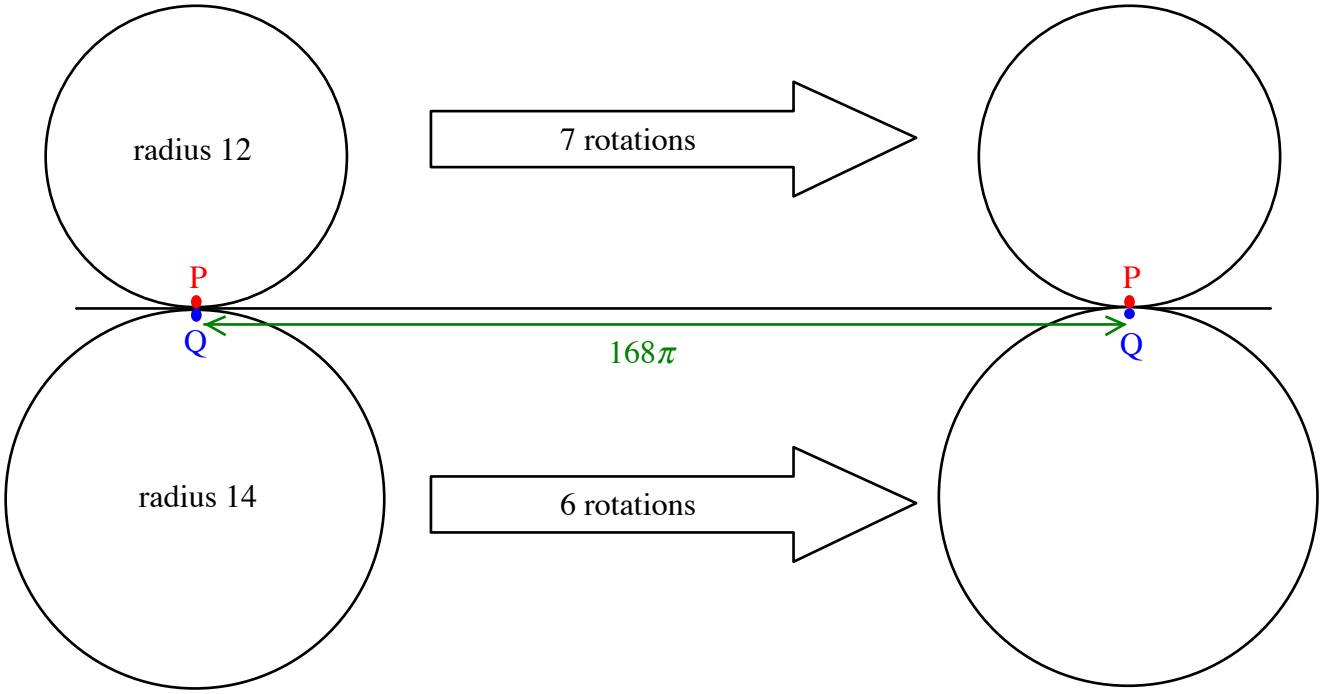
Rotating Disk Puzzle—C.E. Mungan, Spring 2018

Two circular disks have radii of 12 and 14. The larger disk is held fixed (i.e., it it can neither translate nor rotate in the lab frame of reference) while the smaller disk can roll around the outside of the larger disk without slipping relative to it. In their starting positions, point P on the smaller disk coincides with point Q on the larger disk. How many rotations of the smaller disk occur before points P and Q first coincide again?

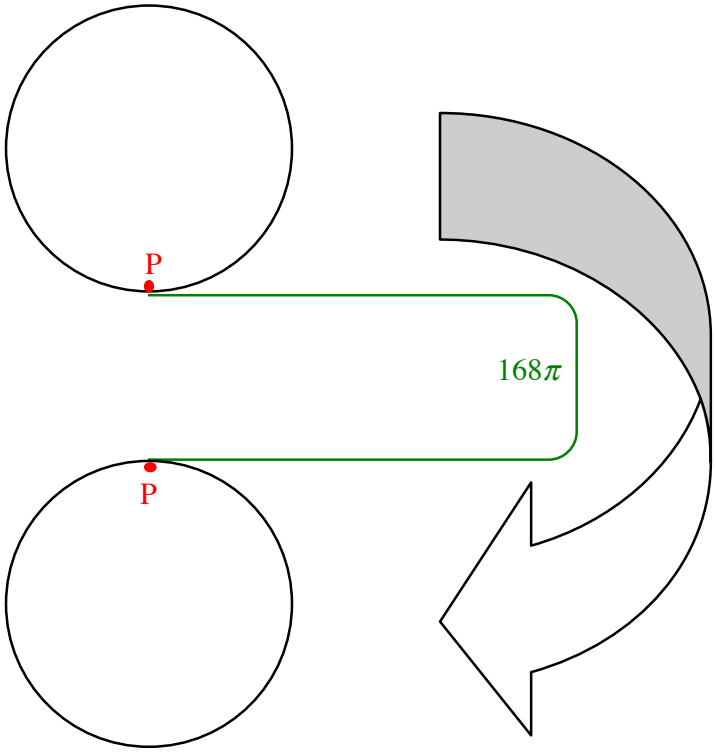
The answer is 13. To see why, first suppose both disks roll without slipping along a straight track separating the two disks, as shown in the following diagram. They will come back into coincidence when the upper disk undergoes 7 rotations and the lower disk 6 rotations. Points P and Q then each have traveled horizontally a net displacement of 168π (which equals $7\times 2\pi\,12$ for the smaller disk or $6\times 2\pi\,14$ for the larger disk) and have thus returned into contact with each other. Note that 7 and 6 have no common factors and so this is the first time that points P and Q come back into coincidence.



Next suppose the upper disk rolls 7 times but this time along a U-shaped track of length 168π as shown on the top of the next page. Clearly the disk now ends up inverted (so that point P is still in contact with the track) and it thus has rotated 7.5 times in space instead of just 7 times.

If the track made a full loop around back to its starting point (but remained of length 168π), then the disk would rotate 8 times in space when traveling along it. It would make 1 extra rotation because the track loops around once in space. As a check on this result, try rolling a penny around the circumference of another penny. Start with the upper penny having heads oriented upright so you can easily tell how many times it has rotated around. By the time the penny gets halfway around the lower penny, it will again have heads upright and will have made

exactly one full rotation in space. When it gets back to its starting position after one circuit around the lower penny, it will thus have made two rotations, i.e., 1 extra rotation because it followed a track that looped around once in space.



Finally, in our problem, our track of length 168π loops 6 times around the lower disk. That results in 6 extra rotations in space of the upper disk traveling along that trajectory, in addition to the 7 rotations we found in the first diagram in this paper, for a total of 13 overall rotations of the smaller disk as measured by a stationary lab observer.