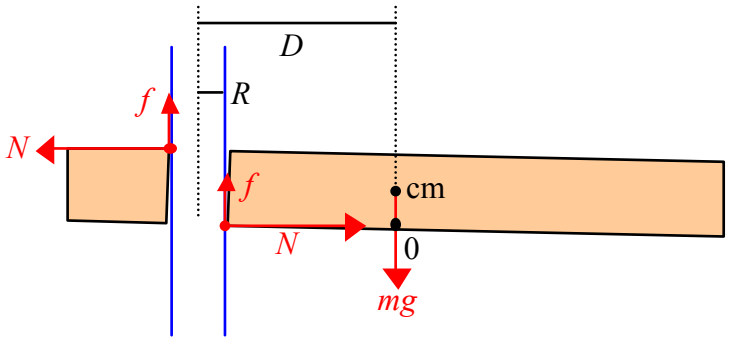
Sliding Platform Held by Static Friction—C.E. Mungan, Spring 2020

A symmetric (such as rectangular or circular) platform has a hole drilled in it with a radius slightly larger than the radius R of a vertical support pole. The hole is off-center a distance D from the center of mass (cm) of the plate. When held horizontally, the platform can be slid along the pole, but when released it tilts slightly so that static friction f holds it at the two points indicated in red below. (The hole can be chamfered slightly so that the contact is at two small areas rather than points.) Find the minimum value of the coefficient of static friction μ to keep the unsupported plate from sliding down the pole.



Let the mass of the platform be m and let the two normal forces at the areas of contact be N. Those two normal forces must be equal and opposite since the plate is in equilibrium. Since the plate is on the verge of slipping, the two friction forces must have their maximum values and thus both must be equal to $f = \mu N$. Now find the point of intersection 0 between the lines of action of the right-pointing normal force N and the gravitational force mg. Balancing torques on the platform about that point implies

$$(D-R)f + (D+R)f = NT \tag{1}$$

where T is the thickness of the plate. This equation simplifies to

$$\boxed{\mu = \frac{T}{2D}} \tag{2}$$

for the minimum value. We need the platform to tilt if there are to be normal forces N and hence holding frictional forces f, and smaller values of T and larger values of D lead to a larger tilt, so that the minimum value of μ is correspondingly decreased.

It may initially seem surprising that Eq. (2) does not depend on the mass of the plate, given that the vertical force balance implies that $2f = mg$ so that the friction depends on m. However, Eq. (1) implies that the ratio of f to N is independent of m. In other words, while we need a larger f to hold a heavier plate, such a heavier plate also increases the normal force proportionally and thus no increase in the coefficient of friction is required. That is good, because it means our platform will not slip even if we place objects on it (as long as we distribute them to keep the center of mass position at least a distance D away from the center of the pole).