Two-Slit Interference Using a Thermal Source of Light—C.E. Mungan, Fall 2016

It is possible to do two-slit interference with blackbody radiation such as sunlight, even without passing that radiation through a pinhole first. For example, see the Veritasium video https://www.youtube.com/watch?v=Iuv6hY6zsd0. Here I briefly explain how the interference arises, because thermal radiation is the prototypical example of incoherent light, whereas standard two-slit demonstrations usually use coherent laser radiation.

The key idea is that if we restrict attention to a single emitting spot on the surface of the thermal radiator (say a point on a hot tungsten filament in a light bulb), then it randomly (i.e., spontaneously) emits short pulses (alternatively called packets or trains) of light. Each pulse is coherent over a time interval Δt that is approximately related to the frequency bandwidth Δf of the pulse as $\Delta t \approx 1/\Delta f$ according to the uncertainty principle. Thus the coherence length L of the pulse of light is $L = c\Delta t \approx c/\Delta f$. By using the Planck formula for blackbody emission, one can show that both the bandwidth Δf and c/λ_{max} where λ_{max} is the peak wavelength (from Wien's law) are proportional to the temperature T of the source with a similar constant of proportionality (of about $4k$ where k is the Boltzmann constant), so that $L \approx \lambda_{\text{max}}$ which is on the order of 1 μm for a white-light thermal source such as an incandescent bulb or the sun [1].

Now suppose the source is lined up directly in front of one of the slits, a distance r from it. Then the photons in that pulse have to travel a distance $(r^2 + d^2)^{1/2}$ to get to the other slit, where d is the distance between the two slits. The difference in pathlengths is thus

$$r\sqrt{1 + d^2/r^2} - r \approx d^2/2r \tag{1}$$

since $d \ll r$. We will get interference provided that difference is less than the coherence length, which happens if

$$\boxed{\frac{d^2}{r\lambda_{\text{max}}} \ll 1} \tag{2}$$

neglecting a factor of 2. (Compare this inequality to the condition $a^2 \ll L\lambda$ for monochromatic light of wavelength λ to pass through a single slit of width a and be in the far-field regime when it reaches a screen at a distance L away [2].)

If $d \approx 10$ μm and $\lambda_{\text{max}} \approx 1$ μm, then Eq. (2) implies that we will get two-slit interference provided the source is at least 100 μm away, which is automatically satisfied for any reasonable thermal source (say a light bulb 1 m away or the sun 10^{11} m away). A more rigorous treatment using the van Cittert-Zernike theorem says the field points merely need to be a few wavelengths away from the source to attain coherence [3]!

Two final comments are worth making. First, it is helpful to use a color filter to reduce the bandwidth Δf of the thermal radiation. Doing so in the Veritasium video, for example, would have both obviated the distracting coloration of the fringes and increased the coherence length. In any case, some filtering already occurs due to the limited responsivity of our eye, so that even a light bulb which emits considerable infrared has an effective bandwidth of only about 100 THz.

Second, how does the size of the emitting source affect things? In the case of the sun, its angular width is about half a degree. That leads to a superposition of interference maxima of a given order spread over that same angle on the viewing screen. Thus, to maintain visibility of the fringes, the maxima need to be spaced farther apart than that. Since their angular spacing is about $\lambda_{\max} / d$ at small angular positions on the screen, we require

$$\frac{\lambda_{\max}}{d} >> 0.5° \frac{\pi \text{ rad}}{180°} \quad \Rightarrow \quad d << 100\lambda_{\max} \tag{3}$$

which explains why I chose a value of $d \approx 10\lambda_{\max}$ for the slit separation in the preceding numerical estimate. This result also implies that if one uses a light bulb, it should be unfrosted and preferably have a linear filament aligned with the slit orientation.

[1] A. Donges, "The coherence length of blackbody radiation," Eur. J. Phys. **19**, 245 (1998).

[2] C.L. Panuski and C.E. Mungan, "Single-slit diffraction: Transitioning from geometric optics to the Fraunhofer regime," Phys. Teach. **54**, 325 (2016).

[3] G.S. Agarwal, G. Gbur, and E. Wolf, "Coherence properties of sunlight," Opt. Lett. **29**, 459 (2004).