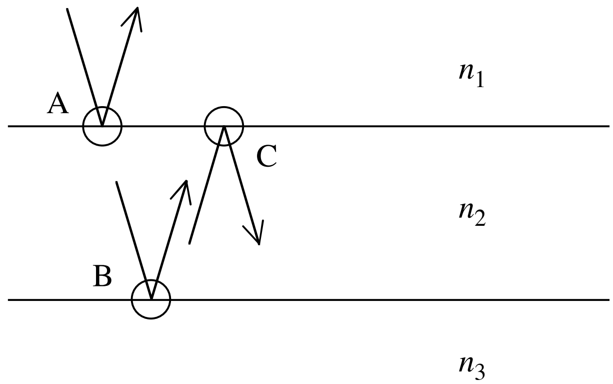
Summary of Thin-Film Interference—C.E. Mungan, Spring 2008

We have three media of indices n_1, n_2, and n_3. For the two dominant reflected and transmitted rays, there are three reflections $\{A, B, C\}$ to consider, as circled below.



For reflected interference, the two reflections of interest are A and B, while in transmittance the two relevant reflections are B and C; in either case, denote the phase changes for these two reflections as ϕ_1 and ϕ_2. Then the interference condition for normal incidence is

$$\boxed{2t = (m + \Delta)\lambda / n_2} \quad \text{with } m = 0, 1, 2, \ldots \tag{1}$$

where λ is the vacuum wavelength and t is the film thickness.[1] The factor Δ accounts for the relative phase change upon reflection. For constructive interference it becomes

$$\Delta = \begin{cases} 0 & \text{if } \phi_1 = \phi_2 \\ \frac{1}{2} & \text{if } \phi_1 \neq \phi_2 \end{cases} \quad \text{TO BE BRIGHT,} \tag{2a}$$

recalling that the phase change ϕ for a reflection is either 0 or π depending on whether the index of the incident medium is respectively larger or smaller than that of the medium off which the ray is reflecting.[2]

Equations (1) and (2) are universal—they work both for reflected and transmitted interference, for any values of the three indices. In contrast, equations in texts typically only hold for special cases (reflection or transmission for particular relative values of the indices).

If we want destructive interference, we simply swap[3] the two conditions in Eq. (2a) to get

$$\Delta = \begin{cases} 0 & \text{if } \phi_1 \neq \phi_2 \\ \frac{1}{2} & \text{if } \phi_1 = \phi_2 \end{cases} \quad \text{TO BE DARK.} \tag{2b}$$

Reflected and transmitted interference have reflection B in common, but only the former has reflection A and only the latter has reflection C with $\phi_A \neq \phi_C$ since the incident and transmitted media are swapped for them. We conclude that a bright fringe in reflectance is always accompanied by a dark fringe in transmittance, and vice versa. This conclusion accords with energy conservation, since whatever light is not reflected from the film must be transmitted by it.

Explanatory endnotes:

1. If $\Delta = 0$, then the case $m = 0$ corresponds to a film whose thickness is much less than the wavelength of light (in the film), such as for the central dark spot in Newton's rings.

2. Equation (2a) is derived by defining $\Delta \equiv |\phi_1 - \phi_2| / 2\pi$ for a bright fringe.

3. Note that swapping the two values of Δ gives the same result as adding a factor of 1/2 to Δ, and then subtracting 1 if $\Delta = 1$ because one extra optical cycle is equivalent to a phase difference of zero.