

Names of the Trigonometric Functions—C.E. Mungan, Fall 2008

The purpose of this article is two-fold: to suggest some simple mnemonics for remembering the definitions of the various trigonometric functions, and to briefly provide some history and etymology of their names.

If one starts by taking sine, cosine and tangent as the fundamental trio of trig functions, then “SOHCAHTOA” (Sine = Opposite/Hypotenuse, Cosine = Adjacent/Hypotenuse, Tangent = Opposite/Adjacent) suffices to remember them. To this mnemonic, one can append “All Students Take Calculus” with words in the order I–II–III–IV to remember that All three functions are positive in Quadrant I, only Sine is positive in Quadrant II, only Tangent is positive in Quadrant III, and only Cosine is positive in Quadrant IV.

To obtain the other three trig functions, I find it useful to remember that a co-function is the reciprocal of a non-co-function (and vice versa):

$$\text{cotangent} = 1/\text{tangent}$$

$$\text{cosine} = 1/\text{secant}$$

$$\text{cosecant} = 1/\text{sine}$$

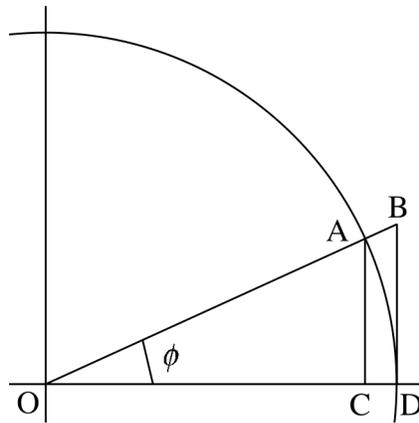
While that rule is a helpful memory trick, it begs the question of why cosecant is not equal to $1/\text{secant}$, and cosine is not equal to $1/\text{sine}$? The answer is that the actual historical reason for the names is that “co” refers to “complementary” so that $\text{co-trig}(\text{angle}) = \text{trig}(\text{co-angle})$:

$$\cot(\theta) = \tan(90^\circ - \theta)$$

$$\cos(\theta) = \sin(90^\circ - \theta)$$

$$\csc(\theta) = \sec(90^\circ - \theta)$$

As this latter rule correctly suggests, the base trio of trig functions are in fact the non-complementary functions (tangent, sine, secant) rather than the set listed in the second paragraph of this article. The name tangent brings to mind the fact that the derivative dy/dx of a function $y(x)$ at some point x is the slope of the *tangent* line (tangent = rise/run) at that point. However, that is not the historical origin of the name, as the trig functions certainly predate calculus! The actual etymology deals with angles for a unit circle. Specifically consider the angle ϕ sketched at the top of the next page. (I have followed the helpful rule of drawing an arbitrary angle that is greater than 0° but less than 45° , so that the values of the various trig functions can be clearly distinguished from one another.) Since the circle is a unit circle, $OA = OD = 1$.



We note that:

$$\tan(\phi) = BD$$

$$\sin(\phi) = AC$$

$$\sec(\phi) = OB$$

The result for tangent makes sense because line *BD* is *tangent* to the circle. The word “sine” derives from the Latin “sinus” (referring in current English to *bends* in the nasal cavity); if we reflect shape *ACD* across the horizontal axis above, we get the shape of a *bent* bow and string. Finally, “secant” comes from the Latin verb “secare” (to cut, as in *scissors*) because line *OB* *cuts* across the circle and tangent line.

References:

<http://mathforum.org/library/drmath/view/52578.html>

<http://www.tricochet.com/math/pdfs/completetriangle.pdf>