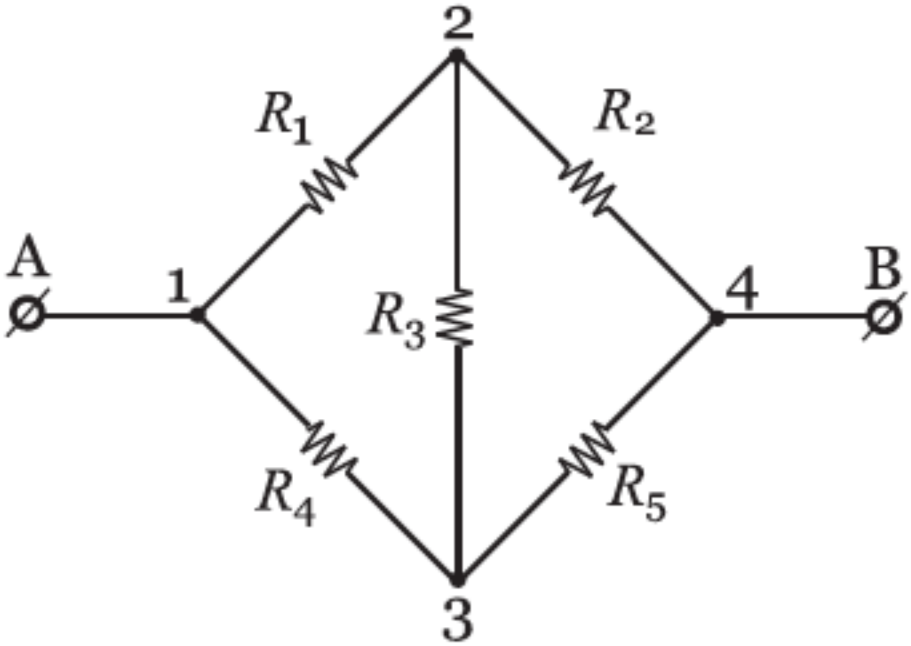
Equivalent Resistance of a Wheatstone Bridge—C.E. Mungan, Fall 2014

*ref: AJP **83**, 53 (2015)*

Find the equivalent resistance of the Wheatstone bridge sketched below.



The method of nodal potentials is simpler than the Kirchhoff method of branch circuits. We assume that point A is connected to the positive terminal of a battery of potential ε, and point B to the negative terminal of zero potential. Let node 2 have potential V_2 and node 3 have potential V_3. Then the junction rule implies that the sum of the currents into node 2 is zero,

$$\frac{\varepsilon - V_2}{R_1} + \frac{V_3 - V_2}{R_3} + \frac{0 - V_2}{R_2} = 0 \tag{1}$$

and likewise for node 3,

$$\frac{\varepsilon - V_3}{R_4} + \frac{V_2 - V_3}{R_3} + \frac{0 - V_3}{R_5} = 0\,. \tag{2}$$

Conductance σ is the reciprocal of resistance R. Defining $\sigma_{123} \equiv \sigma_1 + \sigma_2 + \sigma_3$ and likewise for σ_{345}, and $\upsilon_2 \equiv V_2 / \varepsilon$ and likewise for υ_3, Eqs. (1) and (2) can be rewritten as

$$\sigma_{123}\upsilon_2 - \sigma_3\upsilon_3 = \sigma_1 \tag{3}$$

and

$$-\sigma_3\upsilon_2 + \sigma_{345}\upsilon_3 = \sigma_4\,. \tag{4}$$

The equivalent conductance of the circuit is the current flowing from the positive to the negative terminal of the battery—equal to the sum of the currents across resistors 2 and 5—divided by the battery voltage,

$$\sigma_{\mathrm{eq}} = \frac{(V_2 - 0)/R_2 + (V_3 - 0)/R_5}{\varepsilon} \tag{5}$$

which simplifies to

$$R_{\mathrm{eq}} = \left(\sigma_2 \upsilon_2 + \sigma_5 \upsilon_3\right)^{-1}. \tag{6}$$

Linear equations (3) and (4) are to be solved simultaneously for υ_2 and υ_3, and substituted into this expression to get the final solution.