CHAPTER 11
Rolling, Torque, and Angular Momentum

11-1 ROLLING AS TRANSLATION AND ROTATION COMBINED

Learning Objectives

After reading this module, you should be able to …

11.01 Identify that smooth rolling can be considered as a combination of pure translation and pure rotation.
11.02 Apply the relationship between the center-of-mass speed and the angular speed of a body in smooth rolling.

Key Ideas

• For a wheel of radius $R$ rolling smoothly,

$$v_{com} = \omega R$$

where $v_{com}$ is the linear speed of the wheel's center of mass and $\omega$ is the angular speed of the wheel about its center.

• The wheel may also be viewed as rotating instantaneously about the point $P$ of the “road” that is in contact with the wheel. The angular speed of the wheel about this point is the same as the angular speed of the wheel about its center.

What Is Physics?

As we discussed in Chapter 10, physics includes the study of rotation. Arguably, the most important application of that physics is in the rolling motion of wheels and wheel-like objects. This applied physics has long been used. For example, when the prehistoric people of Easter Island moved their gigantic stone statues from the quarry and across the island, they dragged them over logs acting as rollers. Much later, when settlers moved westward across America in the 1800s, they rolled their possessions first by wagon and then later by train. Today, like it or not, the world is filled with cars, trucks, motorcycles, bicycles, and other rolling vehicles.

The physics and engineering of rolling have been around for so long that you might think no fresh ideas remain to be developed. However, skateboards and inline skates were invented and engineered fairly recently, to become huge
financial successes. Street luge is now catching on, and the self-righting Segway (Fig. 11-1) may change the way people move around in large cities. Applying the physics of rolling can still lead to surprises and rewards. Our starting point in exploring that physics is to simplify rolling motion.

Rolling as Translation and Rotation Combined

Here we consider only objects that roll smoothly along a surface; that is, the objects roll without slipping or bouncing on the surface. Figure 11-2 shows how complicated smooth rolling motion can be: Although the center of the object moves in a straight line parallel to the surface, a point on the rim certainly does not. However, we can study this motion by treating it as a combination of translation of the center of mass and rotation of the rest of the object around that center.

To see how we do this, pretend you are standing on a sidewalk watching the bicycle wheel of Fig. 11-3 as it rolls along a street. As shown, you see the center of mass $O$ of the wheel move forward at constant speed $v_{com}$. The point $P$ on the
street where the wheel makes contact with the street surface also moves forward at speed $v_{\text{com}}$, so that $P$ always remains directly below $O$.

**Figure 11-3**
The center of mass $O$ of a rolling wheel moves a distance $s$ at velocity $v_{\text{com}}$, while the wheel rotates through angle $\theta$. The point $P$ at which the wheel makes contact with the surface over which the wheel rolls also moves a distance $s$.

During a time interval $t$, you see both $O$ and $P$ move forward by a distance $s$. The bicycle rider sees the wheel rotate through an angle $\theta$ about the center of the wheel, with the point of the wheel that was touching the street at the beginning of $t$ moving through arc length $s$. Equation 10-17 relates the arc length $s$ to the rotation angle $\theta$:

$$s = \theta R,$$

(11-1)

where $R$ is the radius of the wheel. The linear speed $v_{\text{com}}$ of the center of the wheel (the center of mass of this uniform wheel) is $\omega R$. The angular speed $\omega$ of the wheel about its center is $\omega$. Thus, differentiating Eq. 11-1 with respect to time (with $R$ held constant) gives us

$$v_{\text{com}} = \omega R \quad \text{(smooth rolling motion)}.$$

(11-2)

**A Combination.** Figure 11-4 shows that the rolling motion of a wheel is a combination of purely translational and purely rotational motions. Figure 11-4a shows the purely rotational motion (as if the rotation axis through the center were stationary): Every point on the wheel rotates about the center with angular speed $\omega$. (This is the type of motion we considered in Chapter 10.) Every point on the outside edge of the wheel has linear speed $v_{\text{com}}$ given by Eq. 11-2. Figure 11-4b shows the purely translational motion (as if the wheel did not rotate at all): Every point on the wheel moves to the right with speed $v_{\text{com}}$.

**Figure 11-4**
Rolling motion of a wheel as a combination of purely rotational motion and purely translational motion. (a) The purely rotational motion: All points on the wheel move with the same angular speed $\omega$. Points on the outside edge of the wheel all move with the speed $v_{\text{com}}$.

(b) The purely translational motion: The center of the wheel moves to the right with speed $v_{\text{com}}$. Points on the outside edge move with speed $v_{\text{com}}$.

(c) The rolling motion: The center of the wheel moves to the right with speed $v_{\text{com}}$. Points on the outside edge move with speed $v_{\text{com}}$.

http://edugen.wileyplus.com/edugen/courses/crs7165/halliday9781118230...XsNzgxMTE4MjMwMCMDAxLnhmb3Jt.enc?course=crs7165&id=ref
same linear speed \( \vec{v} = \vec{v}_{\text{com}} \). The linear velocities \( \vec{v} \) of two such points, at top (\( T \)) and bottom (\( P \)) of the wheel, are shown. (b) The purely translational motion: All points on the wheel move to the right with the same linear velocity \( \vec{v}_{\text{com}} \). The rolling motion of the wheel is the combination of (a) and (b).

The combination of Figs. 11-4a and 11-4b yields the actual rolling motion of the wheel, Fig. 11-4c. Note that in this combination of motions, the portion of the wheel at the bottom (at point \( P \)) is stationary and the portion at the top (at point \( T \)) is moving at speed \( 2v_{\text{com}} \), faster than any other portion of the wheel. These results are demonstrated in Fig. 11-5, which is a time exposure of a rolling bicycle wheel. You can tell that the wheel is moving faster near its top than near its bottom because the spokes are more blurred at the top than at the bottom.

![Figure 11-5](http://edugen.wileyplus.com/edugen/courses/crs7165/halliday9781118230...XksNzg4MTE4MjIwMzI1YTExLXNjY0wMDAxLnhmb3Jt.enc?course=crs7165&id=ref)

_Courtesy Alice Halliday_

**Figure 11-5**

A photograph of a rolling bicycle wheel. The spokes near the wheel's top are more blurred than those near the bottom because the top ones are moving faster, as Fig. 11-4c shows.

The motion of any round body rolling smoothly over a surface can be separated into purely rotational and purely translational motions, as in Figs. 11-4a and 11-4b.

**Rolling as Pure Rotation**

Figure 11-6 suggests another way to look at the rolling motion of a wheel—namely, as pure rotation about an axis that always extends through the point where the wheel contacts the street as the wheel moves. We consider the rolling motion to be pure rotation about an axis passing through point \( P \) in Fig. 11-4c and perpendicular to the plane of the figure. The vectors in Fig. 11-6 then represent the instantaneous velocities of points on the rolling wheel.
Figure 11-6
Rolling can be viewed as pure rotation, with angular speed $\omega$, about an axis that always extends through $P$. The vectors show the instantaneous linear velocities of selected points on the rolling wheel. You can obtain the vectors by combining the translational and rotational motions as in Fig. 11-4.

**Question:** What angular speed about this new axis will a stationary observer assign to a rolling bicycle wheel?

To verify this answer, let us use it to calculate the linear speed of the top of the rolling wheel from the point of view of a stationary observer. If we call the wheel's radius $R$, the top is a distance $2R$ from the axis through $P$ in Fig. 11-6, so the linear speed at the top should be (using Eq. 11-2)

$$v_{\text{top}} = (\omega)(2R) = 2(\omega R) = 2v_{\text{com}}.$$

in exact agreement with Fig. 11-4c. You can similarly verify the linear speeds shown for the portions of the wheel at points $O$ and $P$ in Fig. 11-4c.

**Checkpoint 1**

The rear wheel on a clown's bicycle has twice the radius of the front wheel. (a) When the bicycle is moving, is the linear speed at the very top of the rear wheel greater than, less than, or the same as that of the very top of the front wheel? (b) Is the angular speed of the rear wheel greater than, less than, or the same as that of the front wheel?