Review & Summary

Position
The position $x$ of a particle on an $x$ axis locates the particle with respect to the origin, or zero point, of the axis. The position is either positive or negative, according to which side of the origin the particle is on, or zero if the particle is at the origin. The positive direction on an axis is the direction of increasing positive numbers; the opposite direction is the negative direction on the axis.

Displacement
The displacement $\Delta x$ of a particle is the change in its position:

$$\Delta x = x_2 - x_1.$$  \hspace{1cm} (2-1)

Displacement is a vector quantity. It is positive if the particle has moved in the positive direction of the $x$ axis and negative if the particle has moved in the negative direction.

Average Velocity
When a particle has moved from position $x_1$ to position $x_2$ during a time interval $\Delta t = t_2 - t_1$, its average velocity during that interval is

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}. \hspace{1cm} (2-2)$$

The algebraic sign of $v_{avg}$ indicates the direction of motion ($v_{avg}$ is a vector quantity). Average velocity does not depend on the actual distance a particle moves, but instead depends on its original and final positions.

On a graph of $x$ versus $t$, the average velocity for a time interval $\Delta t$ is the slope of the straight line connecting the points on the curve that represent the two ends of the interval.

Average Speed
The average speed $s_{avg}$ of a particle during a time interval $\Delta t$ depends on the total distance the particle moves in that time interval:

$$s_{avg} = \frac{\text{total distance}}{\Delta t}. \hspace{1cm} (2-3)$$

Instantaneous Velocity
The instantaneous velocity (or simply velocity) $v$ of a moving particle is

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}. \hspace{1cm} (2-4)$$

where $\Delta x$ and $\Delta t$ are defined by Eq. 2-2. The instantaneous velocity (at a particular time) may be found as the slope (at that particular time) of the graph of $x$ versus $t$. Speed is the magnitude of instantaneous velocity.
Average Acceleration

*Average acceleration* is the ratio of a change in velocity \( \Delta v \) to the time interval \( \Delta t \) in which the change occurs:

\[
a_{avg} = \frac{\Delta v}{\Delta t}.
\]  

(2-7)

The algebraic sign indicates the direction of \( a_{avg} \).

Instantaneous Acceleration

*Instantaneous acceleration* (or simply *acceleration*) \( a \) is the first time derivative of velocity \( v(t) \) and the second time derivative of position \( x(t) \):

\[
a = \frac{dv}{dt} = \frac{d^2x}{dt^2}.
\]  

(2-8, 2-9)

On a graph of \( v \) versus \( t \), the acceleration \( a \) at any time \( t \) is the slope of the curve at the point that represents \( t \).

Constant Acceleration

The five equations in Table 2-1 describe the motion of a particle with constant acceleration:

\[
v = v_0 + at,
\]

(2-11)

\[
x - x_0 = v_0t + \frac{1}{2}at^2,
\]

(2-15)

\[
v^2 = v_0^2 + 2a(x - x_0),
\]

(2-16)

\[
x - x_0 = \frac{1}{2}(v_0 + v)t,
\]

(2-17)

\[
x - x_0 = vt - \frac{1}{2}at^2.
\]

(2-18)

These are *not* valid when the acceleration is not constant.

Free-Fall Acceleration

An important example of straight-line motion with constant acceleration is that of an object rising or falling freely near Earth's surface. The constant acceleration equations describe this motion, but we make two changes in notation: (1) we refer the motion to the vertical \( y \) axis with \( \uparrow y \) vertically up; (2) we replace \( a \) with \( -g \), where \( g \) is the magnitude of the free-fall acceleration. Near Earth's surface, \( g = 9.8 \text{ m/s}^2 (= 32 \text{ ft/s}^2) \).