Review & Summary

Position Vector
The location of a particle relative to the origin of a coordinate system is given by a position vector \( \mathbf{r} \), which in unit-vector notation is

\[
\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.
\] (4-1)

Here \( x, y, \) and \( z \) are the vector components of position vector \( \mathbf{r} \), and \( x, y, \) and \( z \) are its scalar components (as well as the coordinates of the particle). A position vector is described either by a magnitude and one or two angles for orientation, or by its vector or scalar components.

Displacement
If a particle moves so that its position vector changes from \( \mathbf{r}_1 \) to \( \mathbf{r}_2 \), the particle's displacement \( \Delta \mathbf{r} \) is

\[
\Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1.
\] (4-2)

The displacement can also be written as

\[
\Delta \mathbf{r} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}
\] (4-3)

\[
= \Delta x\mathbf{i} + \Delta y\mathbf{j} + \Delta z\mathbf{k}.
\] (4-4)

Average Velocity and Instantaneous Velocity
If a particle undergoes a displacement \( \Delta \mathbf{r} \) in time interval \( \Delta t \), its average velocity \( \mathbf{v}_{\text{avg}} \) for that time interval is

\[
\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{r}}{\Delta t}.
\] (4-8)

As \( \Delta t \) in Eq. 4-8 is shrunk to 0, \( \mathbf{v}_{\text{avg}} \) reaches a limit called either the velocity or the instantaneous velocity \( \mathbf{v} \):

\[
\mathbf{v} = \frac{d\mathbf{r}}{dt},
\] (4-10)

which can be rewritten in unit-vector notation as

\[
\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k},
\] (4-11)

where \( v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, \) and \( v_z = \frac{dz}{dt} \). The instantaneous velocity \( \mathbf{v} \) of a particle is always directed along the tangent to the particle's path at the particle's position.

Average Acceleration and Instantaneous Acceleration
If a particle's velocity changes from \( \mathbf{v}_1 \) to \( \mathbf{v}_2 \) in time interval \( \Delta t \), its average acceleration during \( \Delta t \) is
As $\Delta t$ in Eq. 4-15 is shrunk to 0, $\vec{a}_{\text{avg}}$ reaches a limiting value called either the *acceleration* or the *instantaneous acceleration* $\vec{a}$:

$$\vec{a} = \frac{d \vec{v}}{dt}.$$  

(4-16)

In unit-vector notation,

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k},$$  

(4-17)

where $a_x = \frac{dv_x}{dt}$, $a_y = \frac{dv_y}{dt}$, and $a_z = \frac{dv_z}{dt}$.

**Projectile Motion**

*Projectile motion* is the motion of a particle that is launched with an initial velocity $\vec{v}_0$. During its flight, the particle's horizontal acceleration is zero and its vertical acceleration is the free-fall acceleration $-g$ (Upward is taken to be a positive direction.) If $\vec{v}_0$ is expressed as a magnitude (the speed $v_0$) and an angle $\theta_0$ (measured from the horizontal), the particle's equations of motion along the horizontal $x$ axis and vertical $y$ axis are

$$x - x_0 = (v_0 \cos \theta_0) t,$$  

(4-21)

$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} gt^2,$$  

(4-22)

$$v_y = v_0 \sin \theta_0 - gt,$$  

(4-23)

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0).$$  

(4-24)

The trajectory (path) of a particle in projectile motion is parabolic and is given by

$$y = \left(\tan \theta_0\right) x - \frac{g x^2}{2 (v_0 \cos \theta_0)^2}.$$  

(4-25)

if $x_0$ and $y_0$ of Eqs. 4-21 to 4-24 are zero. The particle's **horizontal range** $R$, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$  

(4-26)

**Uniform Circular Motion**

If a particle travels along a circle or circular arc of radius $r$ at constant speed $v$, it is said to be in *uniform circular motion* and has an acceleration $\vec{a}$ of constant magnitude
\[ a = \frac{v^2}{r}. \]  \tag{4-34}

The direction of \( \vec{a} \) is toward the center of the circle or circular arc, and \( \vec{a} \) is said to be \textit{centripetal}. The time for the particle to complete a circle is

\[ T = \frac{2\pi r}{v}. \]  \tag{4-35}

\( T \) is called the \textit{period of revolution}, or simply the \textit{period}, of the motion.

**Relative Motion**

When two frames of reference \( A \) and \( B \) are moving relative to each other at constant velocity, the velocity of a particle \( P \) as measured by an observer in frame \( A \) usually differs from that measured from frame \( B \). The two measured velocities are related by

\[ \vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}, \]  \tag{4-44}

where \( \vec{v}_{BA} \) is the velocity of \( B \) with respect to \( A \). Both observers measure the same acceleration for the particle:

\[ \vec{a}_{PA} = \vec{a}_{PB}. \]  \tag{4-45}