# Review & Summary

## Emf

An **emf device** does work on charges to maintain a potential difference between its output terminals. If \( dW \) is the work the device does to force positive charge \( dq \) from the negative to the positive terminal, then the **emf** (work per unit charge) of the device is

\[
\mathcal{E} = \frac{dW}{dq} \quad \text{(definition of } \mathcal{E}) .
\]

(27-1)

The volt is the SI unit of emf as well as of potential difference. An **ideal emf device** is one that lacks any internal resistance. The potential difference between its terminals is equal to the emf. A **real emf device** has internal resistance. The potential difference between its terminals is equal to the emf only if there is no current through the device.

## Analyzing Circuits

The change in potential in traversing a resistance \( R \) in the direction of the current is \(-IR\); in the opposite direction it is \(+IR\) (resistance rule). The change in potential in traversing an ideal emf device in the direction of the emf arrow is \(+\mathcal{E}\); in the opposite direction it is \(-\mathcal{E}\) (emf rule). Conservation of energy leads to the loop rule:

**Loop Rule.** The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.

Conservation of charge gives us the junction rule:

**Junction Rule.** The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

## Single-Loop Circuits

The current in a single-loop circuit containing a single resistance \( R \) and an emf device with emf \( \mathcal{E} \) and internal resistance \( r \) is

\[
i = \frac{\mathcal{E}}{R + r} ,
\]

which reduces to \( i = \mathcal{E}/R \) for an ideal emf device with \( r = 0 \).

## Power

When a real battery of emf \( \mathcal{E} \) and internal resistance \( r \) does work on the charge carriers in a current \( i \) through the battery, the rate \( P \) of energy transfer to the charge carriers is

\[
P = iV ,
\]

(27-14)

where \( V \) is the potential across the terminals of the battery. The rate \( P_r \) at which energy is dissipated as thermal energy in the battery is

\[
P_r = i^2 r .
\]

(27-16)

The rate \( P_{emf} \) at which the chemical energy in the battery changes is

\[
P_{emf} = i\mathcal{E} .
\]

(27-17)
Series Resistances
When resistances are in **series**, they have the same current. The equivalent resistance that can replace a series combination of resistances is

\[ R_{eq} = \sum_{j=1}^{n} R_j \quad (n \text{ resistances in series}) . \tag{27-7} \]

Parallel Resistances
When resistances are in **parallel**, they have the same potential difference. The equivalent resistance that can replace a parallel combination of resistances is given by

\[ \frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad (n \text{ resistances in parallel}) . \tag{27-24} \]

RC Circuits
When an emf \( \mathcal{E} \) is applied to a resistance \( R \) and capacitance \( C \) in series, as in Fig. 27-15 with the switch at \( a \), the charge on the capacitor increases according to

\[ q = C\mathcal{E}(1 - e^{-t/RC}) \quad (\text{charging a capacitor}) , \tag{27-33} \]

in which \( C\mathcal{E} = q_0 \) is the equilibrium (final) charge and \( RC = \tau \) is the **capacitive time constant** of the circuit. During the charging, the current is

\[ i = \frac{dq}{dt} = \left( \frac{\mathcal{E}}{R} \right) e^{-t/RC} \quad (\text{charging a capacitor}) . \tag{27-34} \]

When a capacitor discharges through a resistance \( R \), the charge on the capacitor decays according to

\[ q = q_0 e^{-t/RC} \quad (\text{discharging a capacitor}) . \tag{27-39} \]

During the discharging, the current is

\[ i = \frac{dq}{dt} = - \left( \frac{q_0}{RC} \right) e^{-t/RC} \quad (\text{discharging a capacitor}) . \tag{27-40} \]