Review & Summary

**LC Energy Transfers**
In an oscillating LC circuit, energy is shuttled periodically between the electric field of the capacitor and the magnetic field of the inductor; instantaneous values of the two forms of energy are

\[ U_E = \frac{q^2}{2C} \quad \text{and} \quad U_B = \frac{L_i^2}{2}, \]

(31-1, 31-2)

where \( q \) is the instantaneous charge on the capacitor and \( i \) is the instantaneous current through the inductor. The total energy \( U(= U_E + U_B) \) remains constant.

**LC Charge and Current Oscillations**
The principle of conservation of energy leads to

\[ L \frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \quad (LC \text{ oscillations}) \]

(31-11)
as the differential equation of LC oscillations (with no resistance). The solution of Eq. 31-11 is

\[ q = Q \cos(\omega t + \phi) \quad (\text{charge}), \]

(31-12)
in which \( Q \) is the charge amplitude (maximum charge on the capacitor) and the angular frequency \( \omega \) of the oscillations is

\[ \omega = \frac{1}{\sqrt{LC}}. \]

(31-4)
The phase constant \( \phi \) in Eq. 31-12 is determined by the initial conditions (at \( t = 0 \)) of the system. The current \( i \) in the system at any time \( t \) is

\[ i = -\omega Q \sin(\omega t + \phi) \quad (\text{current}), \]

(31-13)
in which \( \omega Q \) is the current amplitude \( I \).

**Damped Oscillations**
Oscillations in an LC circuit are damped when a dissipative element \( R \) is also present in the circuit. Then

\[ L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C}q = 0 \quad (RLC \text{ circuit}). \]

(31-24)
The solution of this differential equation is

\[ q = Q e^{-Rt/2L} \cos(\omega't + \phi), \]

(31-25)

where
\[ \omega' = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}. \]  

(31-26)

We consider only situations with small \( R \) and thus small damping; then \( \omega' \approx \omega \).

**Alternating Currents; Forced Oscillations**

A series \( RLC \) circuit may be set into forced oscillation at a driving angular frequency \( \omega_d \) by an external alternating emf

\[ \mathcal{E} = \mathcal{E}_m \sin \omega_d t. \]  

(31-28)

The current driven in the circuit is

\[ i = I \sin(\omega_d t - \phi), \]  

(31-29)

where \( \phi \) is the phase constant of the current.

**Resonance**

The current amplitude \( I \) in a series \( RLC \) circuit driven by a sinusoidal external emf is a maximum \( (I = \mathcal{E}_m/R) \) when the driving angular frequency \( \omega_d \) equals the natural angular frequency \( \omega \) of the circuit (that is, at resonance). Then \( X_C = X_L, \phi = 0 \), and the current is in phase with the emf.

**Single Circuit Elements**

The alternating potential difference across a resistor has amplitude \( V_R = IR \); the current is in phase with the potential difference.

For a **capacitor**, \( V_C = IX_C \), in which \( X_C = 1/\omega_d C \) is the **capacitive reactance**; the current here leads the potential difference by \( 90^\circ \) (\( \phi = -90^\circ = -\pi/2 \) rad).

For an **inductor**, \( V_L = IX_L \), in which \( X_L = \omega_d L \) is the **inductive reactance**; the current here lags the potential difference by \( 90^\circ \) (\( \phi = +90^\circ = +\pi/2 \) rad).

**Series RLC Circuits**

For a series \( RLC \) circuit with an alternating external emf given by Eq. 31-28 and a resulting alternating current given by Eq. 31-29,

\[ I = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} \]  

(31-60, 31-63)

and

\[ \tan \phi = \frac{X_L - X_C}{R} \]  

(31-65)

Defining the impedance \( Z \) of the circuit as

\[ Z = \sqrt{R^2 + (X_L - X_C)^2} \]  

(31-61)
allows us to write Eq. 31-60 as $I = \varepsilon_m/Z$.

### Power

In a series **RLC** circuit, the average power $P_{avg}$ of the generator is equal to the production rate of thermal energy in the resistor:

$$P_{avg} = I_{rms}^2 R = \varepsilon_{rms} I_{rms} \cos \phi.$$  \hspace{1cm} (31-71, 31-76)

Here rms stands for root-mean-square; the rms quantities are related to the maximum quantities by $I_{rms} = I/\sqrt{2}$, $V_{rms} = V/\sqrt{2}$, and $\varepsilon_{rms} = \varepsilon_{m}/\sqrt{2}$. The term $\cos \phi$ is called the power factor of the circuit.

### Transformers

A transformer (assumed to be ideal) is an iron core on which are wound a primary coil of $N_p$ turns and a secondary coil of $N_s$ turns. If the primary coil is connected across an alternating-current generator, the primary and secondary voltages are related by

$$V_s = V_p \frac{N_s}{N_p} \quad \text{(transformation of voltage).}$$ \hspace{1cm} (31-79)

The currents through the coils are related by

$$I_s = I_p \frac{N_p}{N_s} \quad \text{(transformation of currents),}$$ \hspace{1cm} (31-80)

and the equivalent resistance of the secondary circuit, as seen by the generator, is

$$R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R_s$$ \hspace{1cm} (31-82)

where $R$ is the resistive load in the secondary circuit. The ratio $N_p/N_s$ is called the transformer's turns ratio.