I. Types of Waves

A. Mechanical waves. These waves have two central features: They are governed by Newton’s laws, and they can exist only within a material medium, such as water, air, and rock. Common examples include water waves, sound waves, and seismic waves.

B. Electromagnetic waves. These waves require no material medium to exist. All electromagnetic waves travel through a vacuum at the same exact speed $c = 299,792,458$ m/s. Common examples include visible and ultraviolet light, radio and television waves, microwaves, x rays, and radar (We will cover these in Physics II in more detail.)

C. Matter waves. These waves are associated with electrons, protons, and other fundamental particles, and even atoms and molecules. These waves are also called matter waves (study in Quantum Mechanics.)

II. Transverse and Longitudinal Waves

A. In a transverse wave, the displacement of every such oscillating element along the wave is perpendicular to the direction of travel of the wave, as indicated in Fig. below.
B. In a longitudinal wave the motion of the oscillating particles is parallel to the direction of the wave's travel, as shown in Fig. below.

III. Wave variables

A. Transverse Wave Equation

1. The amplitude $y_m$ of a wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as the wave passes through them.

2. The phase of the wave is the argument $(kx - \omega t)$ of the sine function; as the wave sweeps through a string element at a particular position $x$, the phase changes linearly with time $t$.

3. The wavelength $\lambda$ of a wave is the distance parallel to the direction of the wave's travel between repetitions of the shape of the wave (or wave shape). It is related to the angular wave number, $k$, by

4. The period of oscillation $T$ of a wave is the time for an element to move through one full oscillation. It is related to the angular frequency, $\omega$, by

5. The frequency $f$ of a wave is defined as $1/T$ and is related to the angular frequency $\omega$ by

6. A phase constant $\phi$ in the wave function: $y = Y_m \sin(kx - \omega t + \phi)$. The value of $\phi$ can be chosen so that the function gives some other displacement and slope at $x = 0$ when $t = 0$. 
IV. The Speed of a Traveling Wave

A. As the wave in Fig. 16-7 above moves, each point of the moving wave form, such as point A marked on a peak, retains its displacement \( y \). (Points on the string do not retain their displacement, but points on the wave form do.) If point A retains its displacement as it moves, the phase giving it that displacement must remain a constant:

1. 

2. Taking the derivative we get

3. Thus
B. Sample problem: Transverse Wave

1. A wave traveling along a string is described by
   \[ y(x,t) = 0.00327 \sin (72.1x - 2.72t) \]
   in which the numerical constants are in SI units (0.00327 m, 72.1 rad/m, and 2.72 rad/s).

   a) What is the amplitude of this wave?
   
   \[ y_m = \]

   b) What is the wavelength, period, and frequency of this wave?

   c) What is the velocity of this wave?

   d) What is the displacement of the string at \( x = 22.5 \text{ cm} \), and \( t = 18.9 \text{s} \)?
V. \textbf{Wave Speed on a Stretched String}

A. The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig16-8.png}
\caption{A symmetrical pulse, viewed from a reference frame in which the pulse is stationary and the string appears to move right to left with speed \(v\). We find speed \(v\) by applying Newton’s second law to a string element of length \(\Delta l\), located at the top of the pulse.}
\end{figure}

1. A small string element of length \(\Delta l\) within the pulse is an arc of a circle of radius \(R\) and subtending an angle \(2\theta\) at the center of that circle. A force with a magnitude equal to the tension in the string, \(\tau\), pulls tangentially on this element at each end. The horizontal components of these forces cancel, but the vertical components add to form a radial restoring force. For small angles,

2. If \(\mu\) is the linear mass density of the string, and \(\Delta m\) the mass of the small element,

3. The element has acceleration:

4. Therefore,
VI. Energy and Power of a Wave Traveling along a String

A. Transverse speed

B. Kinetic energy

1. \[ dK \]

2. Thus

3. Finally

C. The average power, which is the average rate at which energy of both kinds (kinetic energy and elastic potential energy) is transmitted by the wave, is:
D. Example, Transverse Wave:

1. A string along which waves can travel is 2.70 m long and has a mass of 260g. The tension in the string is 36.0N. What must be the frequency of the traveling waves of amplitude 7.70 mm for the average power to be 85.0 W?

   a) Solution:
VII. The Wave Equation

A. Derivation

1. Let us start with a drawing:

2. If displacement in the y-direction is not absurdly high, then tension is equal on both sides of the string segment. Thus,

3. Then, the net force in the y-direction is $F_y =$

4. Applying NSL:

5. Delta mass:

6. Using the slope of the string segment:
7. Putting it together:

8. Solution of this differential equation takes form:

9. Units of the Constant?

B. Using ____________ we finally get the wave equation: The general differential equation that governs the travel of waves of all types
VIII. The Superposition of Waves

A. The displacement of the string when waves overlap is then the algebraic sum

$$y'(x, t) = y_1(x, t) + y_2(x, t).$$

1. Overlapping waves algebraically add to produce a resultant wave (or net wave).
2. Overlapping waves do not in any way alter the travel of each other.

![Diagram of wave superposition](image.png)
IX. Interference of Waves

A. If two sinusoidal waves of the same amplitude and wavelength travel in the *same* direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.

1. Let us start with two waves:

2. Their algebraic sum:

3. Using Appendix e, the sum of the sines of two angles:

4. Thus their displacement is:

*Fig. 16-12* The resultant wave of Eq. 16-51, due to the interference of two sinusoidal transverse waves, is also a sinusoidal transverse wave, with an amplitude and an oscillating term.
5. **Graphical representation**

![Graphical representation of waves](image)

6. **Table:**

<table>
<thead>
<tr>
<th>Phase Difference, in</th>
<th>Amplitude of Resultant Wave</th>
<th>Type of Interference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degrees</td>
<td>Radians</td>
<td>Wavelengths</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>120</td>
<td>2/3π</td>
<td>0.33</td>
</tr>
<tr>
<td>180</td>
<td>π</td>
<td>0.50</td>
</tr>
<tr>
<td>240</td>
<td>2/3π</td>
<td>0.67</td>
</tr>
<tr>
<td>360</td>
<td>2π</td>
<td>1.00</td>
</tr>
<tr>
<td>865</td>
<td>15.1</td>
<td>2.40</td>
</tr>
</tbody>
</table>

*The phase difference is between two otherwise identical waves, with amplitude y_m, moving in the same direction.*
B. Sample problem:

1. Two identical traveling waves, moving in the same direction, are out of phase by $\pi/2$ rad. What is the amplitude of the resultant wave in terms of the common amplitude $y_m$ of the two combining waves?

   a) Solution:
X. Standing Waves

A. Diagram

As the waves move through each other, some points never move and some move the most.

Fig. 16-16  (a) Five snapshots of a wave traveling to the left, at the times $t$ indicated below part (c) ($T$ is the period of oscillation). (b) Five snapshots of a wave identical to that in (a) but traveling to the right, at the same times $t$. (c) Corresponding snapshots for the superposition of the two waves on the same string. At $t = 0, \frac{1}{2} T$, and $T$, fully constructive interference occurs because of the alignment of peaks with peaks and valleys with valleys. At $t = \frac{1}{4} T$ and $\frac{3}{4} T$, fully destructive interference occurs because of the alignment of peaks with valleys. Some points (the nodes, marked with dots) never oscillate; some points (the antinodes) oscillate the most.

B. If two sinusoidal waves of the same amplitude and wavelength travel in opposite directions along a stretched string, their interference with each other produces a standing wave.

1. Let us start with our two waves

2. Their algebraic sum:
3. Using Appendix e, the sum of the sines of two angles:

4. Thus their displacement is:

5. In the standing wave equation, the amplitude is zero for values of $kx$ that give

   \[ a) \quad \text{Those values are} \]

   \[ b) \quad \text{Since} \]

   \[ , \quad \text{we get} \quad kx = n\pi, \quad n = 0, 1, 2, \ldots \quad \text{(nodes)}, \]

   as the positions of zero amplitude or the nodes.

   The adjacent nodes are therefore separated by

   \[ , \quad \text{half a wavelength.} \]

6. The amplitude of the standing wave has a maximum value of

   \[ , \quad \text{which occurs for values of} \ kx \ \text{that give} \]

   \[ a) \quad \text{Those values are} \]

   \[ b) \quad \text{That is,} \]

   \[ , \quad \text{as the positions of maximum amplitude or the antinodes. The antinodes are separated by} \]

   \[ , \quad \text{and are located halfway between pairs of nodes.} \]
XI. Standing Waves, Reflections at a Boundary

A. Diagram

Fig. 16-18  
(a) A pulse incident from the right is reflected at the left end of the string, which is tied to a wall. Note that the reflected pulse is inverted from the incident pulse.  
(b) Here the left end of the string is tied to a ring that can slide without friction up and down the rod. Now the pulse is not inverted by the reflection.

B. Remember in order to truly solve a differential equation you need to apply your boundary conditions.
XII. Standing Waves and Resonance

A. For certain frequencies, the interference produces a standing wave pattern (or oscillation mode) with nodes and large antinodes like those in Fig. 16-19.

1. Fig. 16-19 Stroboscopic photographs reveal (imperfect) standing wave patterns on a string being made to oscillate by an oscillator at the left end. The patterns occur at certain frequencies of oscillation. (Richard Megna/Fundamental Photographs)

B. Such a standing wave is said to be produced at resonance, and the string is said to resonate at these certain frequencies, called resonant frequencies.
C. The frequencies associated with these modes are often labeled \( f_1, f_2, f_3, \) and so on. The collection of all possible oscillation modes is called the harmonic series, and \( n \) is called the harmonic number of the \( \text{nth} \) harmonic.

![Figure 16-20](image.png)

**Fig. 16-20** A string, stretched between two clamps, is made to oscillate in standing wave patterns. (a) The simplest possible pattern consists of one *loop*, which refers to the composite shape formed by the string in its extreme displacements (the solid and dashed lines). (b) The next simplest pattern has two loops. (c) The next has three loops.

Here \( \nu \) is the speed of traveling waves on the string.
D. Sample problems:

1. A string stretched between two clamps is made to oscillate in standing wave patterns. What is the wavelength for each of the standing patterns shown below if \( L = 100 \text{ cm} \)? What is the harmonic in each case?

\[
\lambda = \ldots \text{ m}, \ldots \text{ harmonic}
\]

2. Strings A and B have identical lengths and linear densities, but string B is under greater tension than string A. Figure below shows four situations, \( (a) \) through \( (d) \), in which standing wave patterns exist on the two strings. In which situations is there the possibility that strings A and B are oscillating at the same resonant frequency?

\[
\lambda = \ldots \text{ m}, \ldots \text{ harmonic}
\]

\( a) \) Answer:
3. A nylon guitar string has a linear density of 7.20 g/m and is under a tension of 150 N. The fixed supports are distance $D = 90.0$ cm apart. The string is oscillating in the standing wave pattern shown in Fig. below. Calculate the (a) speed, (b) wavelength, and (c) frequency of the traveling waves whose superposition gives this standing wave.

\[ D \]

\[ \text{Standing Wave Pattern} \]

\[ a) \quad \text{Solution} \]