I. Electric Potential Energy

A. Experimentally, physicists and engineers discovered that the electric force is conservative and thus has an associated electric potential energy.

B. The motivation for associating a potential energy with a force is that we can then apply the principle of the conservation of mechanical energy to closed systems involving the force.

C. When an electrostatic force acts between two or more charged particles within a system of particles, we can assign an electric potential energy $U$ to the system.

D. If the system changes its configuration from an initial state $i$ to a different final state $f$, the electrostatic force does work $W$ on the particles. If the resulting change is $\Delta U$, then

E. As with other conservative forces, the work done by the electrostatic force is path independent.

F. Usually the reference configuration of a system of charged particles is taken to be that in which the particles are all infinitely separated from one another. The corresponding reference potential energy is usually set be zero. Therefore,
II. Electric Potential:

A. The potential energy per unit charge at a point in an electric field is called the electric potential \( V \) \textit{(or simply the potential)} at that point. This is a scalar quantity.

Thus,

B. The \textit{electric potential difference} \( V \) between any two points \( i \) and \( f \) in an electric field is equal to the difference in potential energy per unit charge between the two points. Thus,

C. The potential difference between two points is thus the negative of the work done by the electrostatic force to move a unit charge from one point to the other.

D. If we set \( U_i = 0 \) at infinity as our reference potential energy, then the electric potential \( V \) must also be zero there. Therefore, the electric potential at any point in an electric field can be defined to be

1. Here \( W_\infty \) is the work done by the electric field on a charged particle as that particle moves in from infinity to point \( f \).

E. The SI unit for potential is the joule per coulomb. This combination is called the \textit{volt (abbreviated V)}.

1. This unit of volt allows us to adopt a more conventional unit for the electric field, \( E \), which is expressed in newtons per coulomb.
F. We can now define an energy unit that is a convenient one for energy measurements in the atomic/subatomic domain: One electron-volt (eV) is the energy equal to the work required to move a single elementary charge $e$, such as that of the electron or the proton, through a potential difference of exactly one volt. The magnitude of this work is $q\Delta V$, and

G. Electric Potential: Work done by an Applied Force

1. If a particle of charge $q$ is moved from point $i$ to point $f$ in an electric field by applying a force to it, the applied force does work $W_{\text{app}}$ on the charge while the electric field does work $W$ on it. The change $K$ in the kinetic energy of the particle is

2. If the particle is stationary before and after the move; Then $K_f$ and $K_i$ are both zero.

3. Relating the work done by our applied force to the change in the potential energy of the particle during the move, one has

4. We can also relate $W_{\text{app}}$ to the electric potential difference $\Delta V$ between the initial and final locations of the particle:
III. Equipotential Surfaces:

A. Adjacent points that have the same electric potential form an equipotential surface, which can be either an imaginary surface or a real, physical surface.

B. No net work $W$ is done on a charged particle by an electric field when the particle moves between two points $i$ and $f$ on the same equipotential surface.

![Diagram of equipotential surfaces]

**Fig. 24-2** Portions of four equipotential surfaces at electric potentials $V_1 = 100 \, V$, $V_2 = 80 \, V$, $V_3 = 60 \, V$, and $V_4 = 40 \, V$. Four paths along which a test charge may move are shown. Two electric field lines are also indicated.

C. Look at and ensure you understand the other examples on page 691.
IV. Calculating the Potential from the Field:

A. Given the below situation, we need to calculate the potential difference between any points i and f in an electric field if we know the field vector all along any path connecting the points.

B. Remember,

Thus for the above situation,

1. Total work:

2.

3. Thus, the potential difference \( V_f - V_i \) between any two points i and f in an electric field is equal to the negative of the line integral from i to f. Since the electrostatic force is conservative, all paths yield the same result.

4. If we set potential \( V_i = 0 \), then

5. This is the potential \( V \) at any point f in the electric field relative to the zero potential at point i. If point i is at infinity, then this is the potential \( V \) at any point f relative to the zero potential at infinity.
V. Potential Due to a Point Charge:

A. A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

B. Consider a point $P$ at distance $R$ from a fixed particle of positive charge $q$. Imagine that we move a positive test charge $q_0$ from point $P$ to infinity. The path chosen can be the simplest one—a line that extends radially from the fixed particle through $P$ to infinity.

C. If $V_f = 0 \ (at \ \infty) \ and \ V_i = V \ (at \ R)$. Then, for the magnitude of the electric field at the site of the test charge,

D. That gives:

1. Switching $R$ to $r$, 

E. Potential Due to a Group of Point Charges

1. The net potential at a point due to a group of point charges can be found with the help of the superposition principle. First the individual potential resulting from each charge is considered at the given point. Then we sum the potentials.

2. For \( n \) charges, the net potential is

F. Sample Problem:

1. The electric field in a region of space has the components \( E_y = E_z = 0 \) and \( E_x = (4.00 \text{ N/C})x \). Point \( A \) is on the \( y \) axis at \( y = 3.00 \text{ m} \), and point \( B \) is on the \( x \) axis at \( x = 4.00 \text{ m} \). What is the potential difference \( V_B - V_A \)?

![Diagram of electric field with points A and B on x and y axes respectively.](image)
VI. Potential Due to an Electric Dipole:

A. Consider the following diagram of a dipole

![Diagram of an electric dipole]

B. At P, the positive point charge (at distance \( r_+ \)) sets up potential \( V_+ \) and the negative point charge (at distance \( r_- \)) sets up potential \( V_- \). Then the net potential at P is:

\[
V = \sum_{i=1}^{2} V_i = V_+ + V_- = \ldots
\]

C. Naturally occurring dipoles are quite small; so we are usually interested only in points that are relatively far from the dipole, such that \( d \ll r \), where \( d \) is the distance between the charges. If \( p = qd \),

\[
r_- - r_+ \approx d \cos \theta \quad \text{and} \quad r_- r_+ \approx r^2.
\]

\( V = \ldots \)
D. Induced Dipole Moment:

Fig. 24-11 (a) An atom, showing the positively charged nucleus (green) and the negatively charged electrons (gold shading). The centers of positive and negative charge coincide. (b) If the atom is placed in an external electric field $\vec{E}$, the electron orbits are distorted so that the centers of positive and negative charge no longer coincide. An induced dipole moment $\vec{p}$ appears. The distortion is greatly exaggerated here.
VII. Potential Due to a Continuous Charge Distribution: Line of Charge

A. Fig. below (a) A thin, uniformly charged rod produces an electric potential $V$ at point $P$. (b) An element can be treated as a particle. (c) The potential at $P$ due to the element depends on the distance $r$. We need to sum the potentials due to all the elements, from the left side (d) to the right side (e).

B. If $\lambda$ is the charge per unit length, then the charge on length $dx$ is:

1. Step 1: $dq =$

2. Step 2: $dV =$

3. Step 3: $dV =$

4. Step 4: $r =$ \( \Rightarrow dV = \)

5. Step 5: $V = \int dV =$


\[
= \frac{\lambda}{4 \pi \varepsilon_0} \left[ \ln \left( x + \left( x^2 + d^2 \right)^{1/2} \right) \right]_0^L
\]

\[
= \frac{\lambda}{4 \pi \varepsilon_0} \left[ \ln \left( L + \left( L^2 + d^2 \right)^{1/2} \right) - \ln d \right].
\]

Remember

\[
= \frac{\lambda}{4 \pi \varepsilon_0} \ln \left[ \frac{L + \left( L^2 + d^2 \right)^{1/2}}{d} \right].
\]

7. Step 7: Replace Lambda with Q/L
VIII. Potential Due to a Continuous Charge Distribution: Charged Disk

A. In Fig. below, consider a differential element consisting of a flat ring of radius \( R' \) and radial width \( dR' \):

B. Its charge has magnitude

\[
 dq = \sigma (2\pi R')(dR')
\]

C. The contribution of this ring to the electric potential at \( P \) is:

\[
 dV = \frac{1}{4\pi \varepsilon_0} \frac{dq}{r} = \frac{1}{4\pi \varepsilon_0} \frac{\sigma (2\pi R')(dR')}{\sqrt{z^2 + R'^2}}
\]

D. The net potential at \( P \) can be found by adding (via integration) the contributions of all the rings from \( R'=0 \) to \( R'=R \):

\[
 V = \int dV = \frac{\sigma}{2\varepsilon_0} \int_0^R \frac{R' dR'}{\sqrt{z^2 + R'^2}} = \frac{\sigma}{2\varepsilon_0} \left( \sqrt{z^2 + R^2} - z \right)
\]

Note: Integral process is \( U \) substitution (See CH-22 page 644)

Let \( U = z^2 + R'^2 \)

\[dU = 2R'dR'\]
IX. Sample problems:

A. The Figure below shows a thin plastic rod of length $L = 14.9$ cm and uniform positive charge $Q = 54.8$ fC lying on an x axis. With $V = 0$ at infinity, find the electric potential at point $P_1$ on the axis, at distance $d = 2.59$ cm from one end of the rod.

1. Solution setup:
B. In Fig. below, a plastic rod having a uniformly distributed charge $Q = -25.6 \text{ pC}$ has been bent into a circular arc of radius $R = 3.71 \text{ cm}$ and central angle $\phi = 120^\circ$. With $V = 0$ at infinity, what is the electric potential at $P$, the center of curvature of the rod?

1. Solution:
C. In Fig. below, a particle of elementary charge $+e$ is initially at coordinate $z = 20$ nm on the dipole axis (here a $z$ axis) through an electric dipole, on the positive side of the dipole. (The origin of $z$ is at the center of the dipole.) The particle is then moved along a circular path around the dipole center until it is at coordinate $z = -20$ nm, on the negative side of the dipole axis. Figure (b) gives the work $W_a$ done by the force moving the particle versus the angle $\theta$ that locates the particle relative to the positive direction of the $z$ axis. The scale of the vertical axis is set by $W_{as} = 4.0 \times 10^{-30}$ J. What is the magnitude of the dipole moment?

1. Solution setup:
X. Calculating the Field from the Potential:

A. Suppose that a positive test charge $q_0$ moves through a displacement from one equipotential surface to the adjacent surface. The work the electric field does on the test charge during the move is $-q_0 \, dV$.

![Diagram of two equipotential surfaces]

B. The work done by the electric field may also be written as the scalar product or $(q_0 \vec{E}) \cdot d\vec{s} = q_0 E \cos \theta \, ds$.

1. Therefore, $-q_0 \, dV = q_0 E \cos \theta \, ds$,

2. That is, $E \cos \theta = -\frac{dV}{ds}$

3. Since $E \cos \theta$ is the component of $E$ in the direction of $ds$,

4. If we take the $s$ axis to be, in turn, the $x$, $y$, and $z$ axes, the $x$, $y$, and $z$ components of $E$ at any point are

5. Therefore, the component of $E$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.
C. Sample problem:

1. In a region of space, the electric potential, in volts, is given by
   \( V = 2xy^2z - 3x^4yz \). What is the magnitude of electric field vector, in N/C, at the point \( \{3, 6, 9\} \)?
XI. Electric Potential Energy of a System of Point Charges:

A. The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

![Figure 24-15](image)

Figure 24-15 shows two point charges \( q_1 \) and \( q_2 \), separated by a distance \( r \). When we bring \( q_1 \) in from infinity and put it in place, we do no work because no electrostatic force acts on \( q_1 \). However, when we next bring \( q_2 \) in from infinity and put it in place, we must do work because \( q_1 \) exerts an electrostatic force on \( q_2 \) during the move.

1. The work done is \( q_2 V \), where \( V \) is the potential that has been set up by \( q_1 \) at the point where we put \( q_2 \).

\[
V = \frac{1}{4 \pi \varepsilon_0} \frac{q_1}{r}
\]

B. Sample problem:

1. A particle of charge +7.5 \( \mu \)C is released from rest at the point \( x = 60 \) cm on an x axis. The particle begins to move due to the presence of a charge \( Q \) that remains fixed at the origin. What is the kinetic energy of the particle at the instant it has moved 40 cm if (a) \( Q = +20 \mu \)C and (b) \( Q = -20 \mu \)C?
XII. Potential of a Charged Isolated Conductor:

A. Remember, an excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor – whether on the surface or inside – come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

\[ V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} \]

B. We know that

C. Since for all points \( E = 0 \) within a conductor, it follows directly that \( V_f = V_i \) for all possible pairs of points \( i \) and \( f \) in the conductor.

Fig. 24-18 (a) A plot of \( V(r) \) both inside and outside a charged spherical shell of radius 1.0 m. (b) A plot of \( E(r) \) for the same shell.
D. Sample problem:

1. Sphere 1 with radius $R_1$ has positive charge $q$. Sphere 2 with radius $2.00R_1$ is far from sphere 1 and initially uncharged. After the separated spheres are connected with a wire thin enough to retain only negligible charge, (a) is potential $V_1$ of sphere 1 greater than, less than, or equal to potential $V_2$ of sphere 2? What fraction of $q$ ends up on (b) sphere 1 and (c) sphere 2? (d) What is the ratio $\sigma_1/\sigma_2$ of the surface charge densities of the spheres?

   a) Solution setup:
XIII. **Isolated Conductor in an Isolated Electric Field:**

A. If an isolated conductor is placed in an external electric field, all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge.

B. The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field that would otherwise be there.

C. Furthermore, the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface. If the conductor in Fig. 24-20 could be somehow removed, leaving the surface charges frozen in place, the internal and external electric field would remain absolutely unchanged.

![Diagram of an uncharged conductor in an external electric field](image)