CH 28

**Magnetic Fields**

I. What Produces Magnetic Field?

A. One way that magnetic fields are produced is to use moving electrically charged particles, such as a current in a wire, to make an electromagnet. The current produces a magnetic field that is utilizable.

B. The other way to produce a magnetic field is by means of elementary particles such as electrons, because these particles have an *intrinsic magnetic field* around them.

1. The magnetic fields of the electrons in certain materials add together to give a net magnetic field around the material. Such addition is the reason why a permanent magnet, has a permanent magnetic field.

2. In other materials, the magnetic fields of the electrons cancel out, giving no net magnetic field surrounding the material.
II. The Definition of B field:

A. We can define a magnetic field, $B$, by firing a charged particle through the point at which is to be defined, using various directions and speeds for the particle and determining the force that acts on the particle at that point. $B$ is then defined to be a vector quantity that is directed along the zero-force axis.

B. The magnetic force on the charged particle, $F_B$, is defined to be:

$$F_B = qv \times B$$

Here $q$ is the charge of the particle, $v$ is its velocity, and $B$ the magnetic field in the region.

C. The magnitude of this force is then:

$$F_B = qv |B| \sin \phi$$

Here $\phi$ is the smallest angle between vectors $v$ and $B$.

D. Finding the Magnetic Force on a Particle:

1. The force $F_B$ acting on a charged particle moving with velocity $v$ through a magnetic Field $B$ is ALWAYS perpendicular to both $v$ and $B$. 

\[ \text{Cross } \vec{v} \text{ into } \vec{B} \text{ to get the new vector } \vec{v} \times \vec{B}. \]
E. The SI unit for $B$ that follows is newton per coulomb-meter per second. For convenience, this is called the tesla (T):

$$1 \text{ T} = \ldots$$

1. An earlier (non-SI) unit for $B$ is the gauss (G), and

$$1 \text{ tesla} = 10^4 \text{ gauss}.$$  

2. Table

<table>
<thead>
<tr>
<th>Table 2B-1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Some Approximate Magnetic Fields</strong></td>
</tr>
<tr>
<td>At surface of neutron star</td>
</tr>
<tr>
<td>Near big electromagnet</td>
</tr>
<tr>
<td>Near small bar magnet</td>
</tr>
<tr>
<td>At Earth's surface</td>
</tr>
<tr>
<td>In interstellar space</td>
</tr>
<tr>
<td>Smallest value in</td>
</tr>
<tr>
<td>magnetically shielded room</td>
</tr>
</tbody>
</table>

F. Sample problem:

1. An alpha particle travels at a velocity $\vec{v}$ of magnitude 550 m/s through a uniform magnetic field $\vec{B}$ of magnitude 0.045 T. (An alpha particle has a charge of $+ 3.2 \times 10^{-19} \text{ C}$ and a mass of $6.6 \times 10^{-27} \text{ kg}$.) The angle between $\vec{v}$ and $\vec{B}$ is 52°. What is the magnitude of (a) the force $\vec{F}_B$ acting on the particle due to the field and (b) the acceleration of the particle due to $\vec{F}_B$? (c) Does the speed of the particle increase, decrease, or remain the same?
Sample problem continued:
III. Magnetic Field Lines:

A. The direction of the tangent to a magnetic field line at any point gives the direction of B at that point.

B. The spacing of the lines represents the magnitude of B — the magnetic field is stronger where the lines are closer together, and conversely.

C. Opposite magnetic poles attract each other, and like magnetic poles repel each other.

D. **No such thing as a Magnetic Monopole** ➔ Every North Pole has an associated South Pole!

E. Field lines emanate from the North Pole and terminate in the South Pole.
IV. Crossed Fields, Discovery of an Electron:

A. Figure

![Diagram of crossed fields](image)

Fig. 28-7 A modern version of J.J. Thomson’s apparatus for measuring the ratio of mass to charge for the electron. An electric field $\vec{E}$ is established by connecting a battery across the deflecting-plate terminals. The magnetic field $\vec{B}$ is set up by means of a current in a system of coils (not shown). The magnetic field shown is into the plane of the figure, as represented by the array of Xs (which resemble the feathered ends of arrows).

B. When the two fields in Fig. 28-7 are adjusted so that the two deflecting forces acting on the charged particle cancel, we have

C. Thus, the crossed fields allow us to measure the speed of the charged particles passing through them.

D. The deflection of a charged particle, moving through an electric field, $E$, between two plates, at the far end of the plates (in the previous problem) is

$$y = \frac{|q|EL^2}{2mv^2} \Rightarrow \frac{m}{|q|} = \frac{B^2L^2}{2vE}$$

Here, $v$ is the particle’s speed, $m$ its mass, $q$ its charge, and $L$ is the length of the plates.
V. Crossed Fields, The Hall Effect:

A. A Hall potential difference \( V \) is associated with the electric field across strip width \( d \), and the magnitude of that potential difference is \( V = Ed \). When the electric and magnetic forces are in balance (Fig. 28-8b),

\[
eE = e v_d B
\]

where \( v_d \) is the drift speed. But,

\[
v_d = \frac{J}{ne} = \frac{i}{neA}
\]

1. Where \( J \) is the current density, \( A \) the cross-sectional area, \( e \) the electronic charge, and \( n \) the number of charges per unit volume.

B. Therefore, here, \( l = (A/d) \), the thickness of the strip.

1. Fig. 28-8 A strip of copper carrying a current \( i \) is immersed in a magnetic field. (a) The situation immediately after the magnetic field is turned on. The curved path that will then be taken by an electron is shown. (b) The situation at equilibrium, which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Thus, the left side is at a higher potential than the right side. (c) For the same current direction, if the charge carriers were positively charged, they would pile up on the right side, and the right side would be at the higher potential.
VI. A Circulating Charged Particle:

A. Consider a particle of charge magnitude $|q|$ and mass $m$ moving perpendicular to a uniform magnetic field $B$, at speed $v$.

B. The magnetic force continuously deflects the particle, and since $B$ and $v$ are always perpendicular to each other, this deflection causes the particle to follow a circular path.

C. The magnetic force acting on the particle has a magnitude of $|q|vB$.

D. For uniform circular motion
E. Diagram

1. Fig. 28-10 Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field, $B$, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force $F_B$; for circular motion to occur, $F_B$ must point toward the center of the circle, (Courtesy John Le P. Webb, Sussex University, England)

![Diagram](image)

VII. Helical Paths:

A. Fig. 28-11 (a) A charged particle moves in a uniform magnetic field, the particle’s velocity $v$ making an angle $\theta$ with the field direction. (b) The particle follows a helical path of radius $r$ and pitch $p$. (c) A charged particle spiraling in a nonuniform magnetic field. (The particle can become trapped, spiraling back and forth between the strong field regions at either end.) Note that the magnetic force vectors at the left and right sides have a component pointing toward the center of the figure.
B. The velocity vector, \( \mathbf{v} \), of such a particle resolved into two components, one parallel to and one perpendicular to it:

\[
\begin{align*}
\text{Pitch} &= \quad \text{and} \quad \text{Radius} = \\
\end{align*}
\]

C. The parallel component determines the pitch \( p \) of the helix (the distance between adjacent turns (Fig. 28-11b)). The perpendicular component determines the radius of the helix.

D. The more closely spaced field lines at the left and right sides indicate that the magnetic field is stronger there. When the field at an end is strong enough, the particle “reflects” from that end. If the particle reflects from both ends, it is said to be trapped in a magnetic bottle.
VIII. Sample problems of Charged Particles in Magnetic Fields

A. Crossed Fields:

1. An electric field of 1.50 kV/m and a perpendicular magnetic field of 0.400 T act on a moving electron to produce no net force. What is the electron's speed?

B. The Hall Effect:

1. A strip of copper 150 μm thick and 4.5 mm wide is placed in a uniform magnetic field \( \mathbf{B} \) of magnitude 0.65 T, with \( \mathbf{B} \) perpendicular to the strip. A current \( i = 23 \text{ A} \) is then sent through the strip such that a Hall potential difference \( V \) appears across the width of the strip. Calculate \( V \). (The number of charge carriers per unit volume for copper is \( 8.47 \times 10^{28} \text{ electrons/m}^3 \).)
C. A Circulating Charged Particle:

1. An electron is accelerated from rest through potential difference $V$ and then enters a region of uniform magnetic field, where it undergoes uniform circular motion. Figure 28-37 gives the radius $r$ of that motion versus $V^{1/2}$. The vertical axis scale is set by $r_s = 3.0$ mm, and the horizontal axis scale is set by $V^{1/2} = 40.0 V^{1/2}$. What is the magnitude of the magnetic field?
IX. Magnetic Force on a Current-Carrying Wire:

A. A force acts on a current carrying wire in a B field:

B. Consider a length $L$ of the wire in the figure. All the conduction electrons in this section of wire will drift past plane $xx$ in a time $t = L/v_d$. 
C. Thus, in that time a charge will pass through that plane that is given by

Here $L$ is a length vector that has magnitude $L$ and is directed along the wire segment in the direction of the (conventional) current.

D. If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments. The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write

and we can find the resultant force on any given arrangement of currents by integrating Eq. 28-28 over that arrangement.

E. Sample problem:

1. A wire 1.80 m long carries a current of 13.0 A and makes an angle of 35.0° with a uniform magnetic field of magnitude $B = 1.50$ T. Calculate the magnetic force on the wire.
X. Torque on a Current Loop:

A. Consider now, a loop of wire with current flowing through it:

B. The two magnetic forces $F$ and $-F$ produce a torque on the loop, tending to rotate it about its central axis.

C. To define the orientation of the loop in the magnetic field, we use a normal vector $n$ that is perpendicular to the plane of the loop. Figure 28-19b shows a right-hand rule for finding the direction of $n$. In Fig. 28-19c, the normal vector of the loop is shown at an arbitrary angle $\theta$ to the direction of the magnetic field.
D. For side 2 the magnitude of the force acting on this side is \( F_2 = ibB \sin(90^\circ - \theta) = ibB \cos \theta = F_4 \). \( F_2 \) and \( F_4 \) cancel out exactly.

E. Forces \( F_1 \) and \( F_3 \) have the common magnitude \( iab \). As Fig. 28-19c shows, these two forces do not share the same line of action; so they produce a net torque.

\[
\tau' = \left( iab \frac{b}{2} \sin \theta \right) + \left( iab \frac{b}{2} \sin \theta \right) = iabB \sin \theta.
\]

F. For \( N \) loops, when \( A = ab \), the area of the loop, the total torque is:

\[
\tau = N \tau' =
\]

G. Sample Problem

1. An electron moves in a circle of radius \( r = 5.29 \times 10^{-11} \) m with speed \( 2.19 \times 10^6 \) m/s. Treat the circular path as a current loop with a constant current equal to the ratio of the electron’s charge magnitude to the period of the motion. If the circle lies in a uniform magnetic field of magnitude \( B = 7.10 \) mT, what is the maximum possible magnitude of the torque produced on the loop by the field?
XI. The Magnetic Dipole Moment, $\mu$:

A. **Definition:**

Here, $N$ is the number of turns in the coil, $i$ is the current through the coil, and $A$ is the area enclosed by each turn of the coil.

B. **Direction:** The direction of $\mu$ is that of the normal vector to the plane of the coil.

C. The definition of torque can be rewritten as:

$$\mu = \text{(magnetic moment)}.$$  

D. Just as in the electric case, the magnetic dipole in an external magnetic field has an energy that depends on the dipole’s orientation in the field:

E. A magnetic dipole has its lowest energy ($-\mu B \cos 0 = -\mu B$) when its dipole moment $\mu$ is lined up with the magnetic field. It has its highest energy ($-\mu B \cos 180^\circ = +\mu B$) when $\mu$ is directed opposite the field.

F. From the below equations, one can see that the unit of $\mu$ can be the joule per tesla (J/T), or the ampere–square meter.
G. Sample Problem:

1. A magnetic dipole with a dipole moment of magnitude 0.020 J/T is released from rest in a uniform magnetic field of magnitude 52 mT. The rotation of the dipole due to the magnetic force on it is unimpeded. When the dipole rotates through the orientation where its dipole moment is aligned with the magnetic field, its kinetic energy is 0.80 mJ. (a) What is the initial angle between the dipole moment and the magnetic field? (b) What is the angle when the dipole is next (momentarily) at rest?