I. Maxwell’s Equations

A. Maxwell’s First Equation

1. In Chapter 23 you learned:

2. 

   a) The integral of the outgoing electric field over an area 
enclosing a volume equals the total charge inside, in appropriate 
units. This can be rewritten as: 
   (Gauss’ Law for Electric Fields)

3. This is Maxwell’s first equation. It represents completely covering the 
surface with a large number of tiny patches having areas. (The little areas are 
small enough to be regarded as flat, the vector magnitude $dA$ is just the value 
of the area, the direction of the vector is perpendicular to the area element, 
pointing outwards away from the enclosed volume.) Hence the dot product 
with the electric field selects the component of that field pointing 
perpendicularly outwards (it would count negatively if the field were pointing 
inwards)—this is the only component of the field that contributes to actual 
electric flux across the surface. (Remember flux just means flow—the picture 
of the electric field in this context is like a fluid flowing out from the charges, 
the field vector representing the direction and velocity of the flowing fluid.)
B. Maxwell’s Second Equation

1. The second Maxwell equation is the analogous one for the magnetic field, which has no sources or sinks (no magnetic monopoles, the field lines just flow around in closed curves). Again thinking of the force lines as representing a kind of fluid flow, the so-called "magnetic flux", we see that for a closed surface, as much magnetic flux flows into the surface as flows out—since there are no sources. This can perhaps be visualized most clearly by taking a group of neighboring lines of force forming a slender tube—the "fluid" inside this tube flows round and round, so as the tube goes into the closed surface then comes out again (maybe more than once) it is easy to see that what flows into the closed surface at one place flows out at another. Therefore the net flux out of the enclosed volume is zero, Maxwell’s second equation:

   a) (Gauss' Law for Magnetic Fields)

   b) ____________________________________________________________________________
   ______________________________________________________________________________.

   c) ____________________________________________________________________________
   ______________________________________________________________________________
   ______________________________________________________________________________.

   d) Some grand unified theories predict the existence of
   ______________________________________________________________________________.

   e) The ________________ theory of magnetic charge started with a paper by the physicist Paul A.M. Dirac in 1931. In this paper, Dirac showed that if any magnetic monopoles exist in the universe, then all electric charge in the universe must be _________________. The electric charge is, in fact, quantized, which suggests (but does not necessarily prove) that monopoles exist.
2. Gauss’ law for magnetic fields holds for structures even if the Gaussian surface does not enclose the entire structure. Gaussian surface II near the bar magnet of Fig. 32-4 encloses no poles, and we can easily conclude that the net magnetic flux through it is zero. For Gaussian surface I, it may seem to enclose only the north pole of the magnet because it encloses the label N and not the label S. However, a south pole must be associated with the lower boundary of the surface because magnetic field lines enter the surface there. Thus, Gaussian surface I encloses a magnetic dipole, and the net flux through the surface is zero.

C. Maxwell’s Third Equation

1. Faraday’s law of Induction from Chapter 30

   a) 

   b) *It relates the induced electric field to the changing magnetic flux.*

   c) *Maxwell theorized that this can work in the opposite; that a changing electrical field can induce a magnetic field.*

D. Maxwell’s Fourth Equation

1. Induced Magnetic Fields

   a) *Maxwell discovered that this can work in the opposite sense...i.e. a changing electrical flux can induce a magnetic field.*

   (1)

   (2) Here \(B\) is the magnetic field induced along a closed loop by the changing electric flux \(\Phi E\) in the region encircled by that loop.
(3) Notice the two equations (32-2 and 32-3) are similar. Sign difference is just example of Lenz’s law. The other difference ________________ is SI Unit conversion.

(4) Fig. 32-5 (a) A circular parallel-plate capacitor, shown in side view, is being charged by a constant current i. (b) A view from within the capacitor, looking toward the plate at the right in (a). The electric field is uniform, is directed into the page (toward the plate), and grows in magnitude as the charge on the capacitor increases. The magnetic field induced by this changing electric field is shown at four points on a circle with a radius r less than the plate radius R.

2. Building Ampere-Maxwell Law

a) Remember Ampere’s Law from CH29

(1)

(2) Here \( i_{\text{enc}} \) is the current encircled by the closed loop

b) Maxwell discovered that the Magnetic Field \( \mathbf{B} \) produced by means other than a magnetic material (that is, by a current and by a changing electric field) give the field in exactly the same form. Thus he combined his law of induction and Ampere’s law together to make a universal equation is not limited to situations where there is no change in electric flux.
c) Notice: When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of the second equation is zero, and so it reduces to the first equation, Ampere’s law.

![Diagram](image)

Fig. 32-6 A uniform magnetic field $\mathbf{B}$ in a circular region. The field, directed into the page, is increasing in magnitude. The electric field $\mathbf{E}$ induced by the changing magnetic field is shown at four points on a circle concentric with the circular region.

E. Displacement Current

1.
2. Comparing the last two terms on the right side of the above equation shows that the term \[ \text{________________________________________} \] must have the dimension of a current. This product is usually treated as being a fictitious current called the displacement current \( i_d \). Therefore we can rewrite as:

\[
\text{(Eq 32-11), in which } i_{d,\text{enc}} \text{ is the displacement current that is encircled by the integration loop } \text{______________________________} \text{ (displacement current)}
\]

3. The charge \( q \) on the plates of a parallel plate capacitor at any time is related to the magnitude \( E \) of the field between the plates at that time by\[ \text{_______________________________________________________} \] in which \( A \) is the plate area.

\[
a)
\]

4. The associated magnetic field are:
F. Maxwell’s Equations

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss’ law for electricity</td>
<td>( \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0 )</td>
<td>Relates net electric flux to net enclosed electric charge</td>
</tr>
<tr>
<td>Gauss’ law for magnetism</td>
<td>( \oint \vec{B} \cdot d\vec{A} = 0 )</td>
<td>Relates net magnetic flux to net enclosed magnetic charge</td>
</tr>
<tr>
<td>Faraday’s law</td>
<td>( \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} )</td>
<td>Relates induced electric field to changing magnetic flux</td>
</tr>
<tr>
<td>Ampere–Maxwell law</td>
<td>( \oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0\varepsilon_\text{m} )</td>
<td>Relates induced magnetic field to changing electric flux and current</td>
</tr>
</tbody>
</table>

*Written on the assumption that no dielectric or magnetic materials are present.*

G. Example Problems:

33. The following is not one of Maxwell’s equations

A. \( \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \)

B. \( \oint \vec{B} \cdot d\vec{A} = 0 \)

C. \( \oint \vec{E} \cdot d\vec{A} = \Phi_C \)

D. \( \oint \vec{B} \cdot d\vec{s} = \mu_0 J + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \)

E. \( \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} \)

34. Which of the following statements is false according to Maxwell’s equations?

A. A changing electric field produces a magnetic field.

B. A changing magnetic field produces an electric field.

C. The net flux of the electric field through a closed surface is proportional to the amount of charge enclosed within the surface.

D. The net flux of the magnetic field through a closed surface is proportional to the amount of charge enclosed within the surface.

E. The equations incorporate the experimental observation that there are no magnetic monopoles in nature.

34. An electric field of magnitude 200 V/m and directed into the page is confined to a circular area of radius 0.2m. The field is increasing at a rate of 15 V/(m-s). The displacement current associated with the electric field is closest to

A. \( 1.7 \times 10^{-11} \) A into the page.

B. \( 3.3 \times 10^{-11} \) A into the page.

C. \( 5.3 \times 10^{-11} \) A out of the page.

D. \( 7.0 \times 10^{-11} \) A out of the page.

E. 0.
H. Additional Sample Problems

1. Figure below shows a closed surface. Along the flat top face, which has a radius of 2.0 cm, a perpendicular magnetic field $\mathbf{B}$ of magnitude 0.30 T is directed outward. Along the flat bottom face, a magnetic flux of 0.70 mWb is directed outward. What are the (a) magnitude and (b) direction (inward or outward) of the magnetic flux through the curved part of the surface?

![Diagram of a closed surface showing a magnetic field and magnetic flux]

2. A parallel plate capacitor with circular plates of radius $R=55$ mm is being charged. Evaluate the field magnitude $B$ for $r=11$ mm and $\frac{dE}{dt} = 1.50 \times 10^{12} \text{ V/(m\cdot s)}$.

   a) What if $r>R$, how does the equation for $B$ change?