CH 35

Interference

I. Optical Interference:

A. Interference of light waves, applied in many branches of science.

B. The blue of the top surface of a *Morpho* butterfly wing is due to optical interference and shifts in color as your viewing perspective changes. *(Philippe Colombi/PhotoDisc//Getty Images)*

II. Light as a Wave:

A. Huygen’s Principle:

1. All points on a wavefront serve as point sources of spherical secondary wavelets. After a time $t$, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.
B. Light as a Wave, Law of Refraction:

1. The law of refraction still applies:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]  
(law of refraction),

2. Proof:

\[ \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}, \quad \sin \theta_1 = \frac{\lambda_1}{hc} \quad \text{(for triangle hce)} \]
\[ \sin \theta_2 = \frac{\lambda_2}{hc} \quad \text{(for triangle hcg)} \]

\[ \frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}, \quad n_1 = \frac{c}{v_1} \quad \text{and} \quad n_2 = \frac{c}{v_2}. \]

\[ \frac{\sin \theta_1}{\sin \theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1} \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]  
(law of refraction),

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**Fig. 35-3** The refraction of a plane wave at an air—glass interface, as portrayed by Huygens’ principle. The wavelength in glass is smaller than that in air. For simplicity, the reflected wave is not shown. Parts (a) through (c) represent three successive stages of the refraction.
C. Light as a Wave, Wavelength and Law of Refraction:

\[ \lambda_n = \lambda \frac{v}{c}, \quad \lambda_n = \frac{\lambda}{n}. \]

\[ f_n = \frac{v}{\lambda_n}. \]

\[ f_n = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f, \]

1. The phase difference between two light waves can change if the waves travel through different materials having different indexes of refraction.

2. To find their new phase difference in terms of wavelengths, we first count the number \( N_1 \) of wavelengths there are in the length \( L \) of medium 1.

\[ \lambda_{n1} = \lambda/n_1; \quad N_1 = \frac{L}{\lambda_{n1}} = \frac{L n_1}{\lambda}. \]

3. Similarly, for medium 2,

\[ N_2 = \frac{L}{\lambda_{n2}} = \frac{L n_2}{\lambda}. \]

4. \[ N_2 - N_1 = \]

\[ n_2 \]

\[ L \]

\[ n_1 \]
D. Light as a Wave, Rainbows and Optical Interference

1. Light waves pass into a water drop along the entire side that faces the Sun. Different parts of an incoming wave will travel different paths within the drop.

2. That means waves will emerge from the drop with different phases. Thus, we can see that at some angles the emerging light will be in phase and give constructive interference.

3. The rainbow is the result of such constructive interference.

![Rainbow Diagram](image)

E. Sample Problems:

1. In the figure below, assume that the two light waves, of wavelength 620 nm in air, are initially out of phase by $\pi$ rad. The indexes of refraction of the media are $n_1 = 1.45$ and $n_2 = 1.65$. What are the (a) smallest and (b) second smallest value of $L$ that will put the waves exactly in phase once they pass through the two media?
2. A laser beam with a wavelength and frequency in air of 540 nm and $5.6 \times 10^{14}$ Hz enters a fluid with refractive index 1.3 at an angle of 40° with respect to the normal to the surface. The frequency and wavelength of the light in the fluid are closest to

A. $5.6 \times 10^{14}$ Hz and 415 nm.
B. $5.6 \times 10^{14}$ Hz and 700 nm.
C. $7.3 \times 10^{14}$ Hz and 540 nm.
D. $4.3 \times 10^{14}$ Hz and 540 nm.
E. $5.6 \times 10^{14}$ Hz and 347 nm.
III. Diffraction:

A. If a wave encounters a barrier that has an opening of dimensions similar to the wavelength, the part of the wave that passes through the opening will flare (spread) out—will diffract—into the region beyond the barrier. The flaring is consistent with the spreading of wavelets according to Huygens principle. Diffraction occurs for waves of all types.

B. A wave passing through a slit flares (diffracts).

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Fig. 35-6 Waves produced by an oscillating paddle at the left flare out through an opening in a barrier along the water surface. (Runk Schoenberger/Grant Heitman Photography)

Fig. 35-7 Diffraction represented schematically. For a given wavelength $\lambda$, the diffraction is more pronounced the smaller the slit width $a$. The figures show the cases for (a) slit width $a = 6.0\lambda$, (b) slit width $a = 3.0\lambda$, and (c) slit width $a = 1.5\lambda$. In all three cases, the screen and the length of the slit extend well into and out of the page, perpendicular to it.
C. Young’s Interference Experiment:

The waves emerging from the two slits overlap and form an interference pattern.

Fig. 35-8  In Young’s interference experiment, incident monochromatic light is diffracted by slit $S_0$, which then acts as a point source of light that emits semicircular wavefronts. As that light reaches screen $B$, it is diffracted by slits $S_1$ and $S_2$, which then act as two point sources of light. The light waves traveling from slits $S_1$ and $S_2$ overlap and undergo interference, forming an interference pattern of maxima and minima on viewing screen $C$. This figure is a cross section: the screens, slits, and interference pattern extend into and out of the page. Between screens $B$ and $C$, the semicircular wavefronts centered on $S_2$ depict the waves that would be there if only $S_2$ were open. Similarly, those centered on $S_1$ depict waves that would be there if only $S_1$ were open.

Fig. 35-9  A photograph of the interference pattern produced by the arrangement shown in Fig. 35-8, but with short slits. (The photograph is a front view of part of screen $C$.) The alternating maxima and minima are called interference fringes (because they resemble the decorative fringe sometimes used on clothing and rugs). (Jearl Walker)
1. The phase difference between two waves can change if the waves travel paths of different lengths.

2. What appears at each point on the viewing screen in a Young’s double-slit interference experiment is determined by the length difference $\Delta L$ of the rays reaching that point.

3. For a bright fringe, $\Delta L$ must be either zero or an integer number of wavelengths. Therefore,

4. For a dark fringe, $\Delta L$ must be an odd multiple of half a wavelength. Therefore,

5. 

\[ \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright fringes).} \]

\[ \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark fringes).} \]
D. Sample Problems:

1. Suppose that Young's experiment is performed with blue-green light of wavelength 500 nm. The slits are 1.20 mm apart, and the viewing screen is 5.40 m from the slits. How far apart are the bright fringes near the center of the interference pattern?
2. A Young’s interference experiment is performed with monochromatic light of wavelength 632.8 nm. In the interference pattern on a screen 3.0m away from the slits, the second minimum is 5 mm away from the center of the pattern. The separation between the slits is closest to

A. 1.21 mm.  
B. 0.57 mm.  
C. 0.76 mm.  
D. 0.99 mm.  
E. 0.36 mm.
3. Monochromatic light of wavelength 537 nm strikes a screen containing 2 slits that are 5.0 μm apart and 2.0 m from a viewing screen. What is the distance on the screen from the center of the interference pattern to the second order maximum?

![Diagram of double-slit experiment]

4. A double-slit arrangement produces interference fringes for sodium light (λ = 589 nm) that are 0.20° apart. What is the angular fringe separation if the entire arrangement is immersed in water (n = 1.33)?
E. Coherence:

1. For the interference pattern to appear on viewing screen C in the figure, the light waves reaching any point P on the screen must have a phase difference that does not vary in time. When the phase difference remains constant, the light from slits $S_1$ and $S_2$ is said to be completely coherent.

2. If the light waves constantly change in time, then the light is said to be incoherent.
IV. Interference from Thin Films:

A. The interference depends on the reflections and the path lengths:

![Diagram showing interference from thin films](image)

**Fig. 35-15** Light waves, represented with ray $i$, are incident on a thin film of thickness $L$ and index of refraction $n_2$. Rays $r_1$ and $r_2$ represent light waves that have been reflected by the front and back surfaces of the film, respectively. (All three rays are actually nearly perpendicular to the film.) The interference of the waves of $r_1$ and $r_2$ with each other depends on their phase difference. The index of refraction $n_1$ of the medium at the left can differ from the index of refraction $n_3$ of the medium at the right, but for now we assume that both media are air, with $n_1 = n_3 = 1.0$, which is less than $n_2$.

B. The phase difference between two waves can change if one or both waves are reflected.

![Diagram showing phase changes](image)

**Fig. 35-16** Phase changes when a pulse is reflected at the interface between two stretched strings of different linear densities. The wave speed is greater in the lighter string. (a) The incident pulse is in the denser string. (b) The incident pulse is in the lighter string. Only here is there a phase change, and only in the reflected wave.
C. Interference from Thin Films, Reflection Phase Shifts:

1. For light, when an incident wave traveling in the medium of greater index of refraction $n$ is reflected at the interface separating the second medium of smaller refractive index, the reflected wave does not undergo a change in phase; that is, its reflection phase shift is zero.

2. When a wave traveling in a medium of smaller index of refraction is reflected at the interface separating the second medium of a higher refractive index, the phase change is $\pi$ rad, or half a wavelength.

D. Given the below drawing:

1. At point $a$ on the front interface, the incident wave (in air) reflects from the medium having the higher of the two indexes of refraction; so the wave of reflected ray $r_1$ has its phase shifted by 0.5 wavelength.

2. At point $b$ on the back interface, the incident wave reflects from the medium (air) having the lower of the two indexes of refraction; the wave reflected there is not shifted in phase by the reflection, and thus neither is the portion of it that exits the film as ray $r_2$.

3. If the waves of $r_1$ and $r_2$ are to be exactly in phase so that they produce fully constructive interference, the path length $2L$ must cause an additional phase difference of 0.5, 1.5, 2.5, ...wavelengths.

$$2L = \frac{\text{odd number}}{2} \times \lambda_{n2} \quad (\text{in-phase waves}).$$
4. If, instead, the waves are to be exactly out of phase so that there is fully destructive interference, the path length $2L$ must cause either no additional phase difference or a phase difference of $1, 2, 3, \ldots$ wavelengths.

$$2L = \text{integer} \times \lambda_{n2} \quad \text{(out-of-phase waves).}$$

But

$$\lambda_{n2} = \frac{\lambda}{n_2},$$

5. Therefore:

E. Interference from Thin Films, Equations Summary:

Note: You must draw the thin film diagram to figure out which equation to use!!!!

<table>
<thead>
<tr>
<th>Normal incidence</th>
<th>No phase shift or both have $\pi$-shift</th>
<th>One of the two wave has $\pi$-shift</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>m=0,1,2,3,...</td>
</tr>
</tbody>
</table>
F. Interference from Thin Films, Film thickness much less than \( \lambda \):

![Image of a soap film](image.png)

**Fig. 35-18** The reflection of light from a soapy water film spanning a vertical loop. The top portion is so thin that the light reflected there undergoes destructive interference, making that portion dark. Colored interference fringes, or bands, decorate the rest of the film but are marred by circulation of liquid within the film as the liquid is gradually pulled downward by gravitation. *(Richard Megna/Fundamental Photographs)*

G. Sample Problems (including practice drawing thin film diagram):

1. Light of wavelength 624 nm is incident perpendicularly on a soap film \((n = 1.33)\) suspended in air. What are the (a) least and (b) second least thicknesses of the film for which the reflections from the film undergo fully constructive interference?
Sample Problems (continued):

2. A plane wave of monochromatic light is incident normally on a uniform thin film of oil that covers a glass plate. The wavelength of the source can be varied continuously. Fully destructive interference of the reflected light is observed for wavelengths of 500 and 700 nm and for no wavelengths in between. If the index of refraction of the oil is 1.30 and that of the glass is 1.50, find the thickness of the oil film.
Sample Problems (continued):

3. In the figure below, two microscope slides touch at one end and are separated at the other end. When light of wavelength 500 nm shines vertically down on the slides, an overhead observer sees an interference pattern on the slides with the dark fringes separated by 1.2 mm. What is the angle between the slides?
V. Michelson's Interferometer:

A. Experiment:

B. If the material has thickness $L$ and index of refraction $n$, then the number of wavelengths along the light’s to-and-fro path through the material is

$$N_m = \frac{2L}{\lambda_n} = \frac{2Ln}{\lambda}.$$  

C. The number of wavelengths in the same thickness $2L$ of air before the insertion of the material is

$$N_a = \frac{2L}{\lambda}.$$  

D. When the material is inserted, the light returned from mirror $M_1$ undergoes a phase change (in terms of wavelengths) of

$$N_m - N_a = \frac{2Ln}{\lambda} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(n - 1).$$
E. For each phase change of one wavelength, the fringe pattern is shifted by one fringe. Thus, by counting the number of fringes through which the material causes the pattern to shift, one can determine the thickness $L$ of the material in terms of $l$.

F. Sample Problem:

1. If mirror $M_2$ in a Michelson interferometer (Fig. 35-21) is moved through 0.233 mm, a shift of 792 bright fringes occurs. What is the wavelength of the light producing the fringe pattern?