Uncertainty “Rules of Thumb”

This is a quick and dirty guide to computing uncertainties from measurements in the SP211 lab. In lab, you measure some quantities (which are subject to experimental uncertainty) and use them to compute other quantities (which are uncertain because they depend on the measurements.)

There are two types of error, which affect the accuracy and precision of your experimental results. Making an analogy with target shooting, accuracy is your ability for a single measurement to be true (like aiming for and striking the bulls-eye) and precision is your ability to get a repeated result in multiple measurements (like having good grouping).

In measurements there are two types of uncertainty: Systematic errors are errors you make or which are inherent in the experiment which keep you from getting an accurate result, while random uncertainties cause repeated measurements to not give the same value. In lab, we compute with the random uncertainties and worry about the effects of systematic errors. We compute uncertainty as a way of making sure we can “can trust” our answer when we compare different results. If the results agree within the uncertainty, we can relax a bit but if they disagree more than the uncertainty, we have to start considering the effects of systematic errors on our experiment.

Uncertainties, regardless of type, are written as: “quantity ± uncertainty” (e.g. 20.03 ± 0.01 g)

We use uncertainties in the following manner: When we compare two results of 1.15 meters to 1.19 meters they are not numerically equal. However, when we compare 1.15 ± 0.05 meters to 1.19 ± 0.05 meters we would conclude that the two values agree within their uncertainties.

This guide is quick and dirty and is NOT the absolutely best way to do this. Your instructor may choose to do a better job than this (and they should, so heed their guidance over this guide).

Quick Estimation of Random Uncertainties

1) **Gauges/Rulers:** You should be able to read the scales to at least ¼ of the smallest graduated markings on the instrument. Estimate the uncertainty as ¼ of the smallest marking on the instrument (rounding up for significant figures). As an example: consider making a measurement with a ruler which is graduated in millimeters. You get a value of 14.3 millimeters where you had to estimate the “.3” part. The uncertainty on the measurement using this rule is 0.25 mm, rounded to 0.3 mm. You'd record: “14.3 ± 0.3 mm” for the value.

2) **Digital Devices** (Multimeters/Scales): Assuming the builder did their job right, the random uncertainty is smaller than the smallest digit displayed, so use +/- 1 of that digit (e.g. mass=20.03 g, uncertainty is +/- 0.01 g). Digital devices are HIGHLY susceptible to calibration (systematic) errors and so systematic errors can dominate the number you read.

   Also, if the digital display fluctuates, the random uncertainty is ½ the full range of fluctuation (e.g. if the displayed number ranges from 19.99 g to 20.07 g, the uncertainty is 0.04 g).

3) **LoggerPro collected data:** Measure something that should be constant. You see the collected data varies and can even use the stat button to get more information. The random uncertainty can be estimated as ½ of the full range of measured values.
**Calculating Uncertainty Using Random uncertainties (Propagating uncertainty)**

Measurements combine to calculate other quantities. Each measurement has an associated random uncertainty so there is an uncertainty on the calculated quantity as well. Here is one technique to “propagate” measurement uncertainties through a calculation to get the uncertainty on the result.

There are three basic steps to computing the uncertainty $\Delta A$ on quantity $A$. Assume that $A$ is defined by the equation

$$ A = \frac{BC}{D} $$

and that (with uncertainties) our measured values are $B \pm \Delta B$, $C \pm \Delta C$ and $D \pm \Delta D$

1) Compute $A$ straight up, assuming no uncertainty in any parameter, with the relevant formula.

2) Now look at the uncertainties in each quantity in the formula. Add or subtract the random uncertainties from $B$, $C$, or $D$ to make $A$ as large as it could possibly get (call it $A_{\text{max}}$). To make $A_{\text{max}}$ as large as possible, here is how you would apply the random errors:

$$ A_{\text{max}} = \frac{(B + \Delta B)(C + \Delta C)}{(D - \Delta D)} $$

3) The uncertainty on $A$ is estimated as the difference between $A$ and $A_{\text{max}}$, ignoring the sign:

$$ \Delta A = | A_{\text{max}} - A | $$