HOW TO WORK COLLISION PROBLEMS

If given information about objects after the collision, then simply use:

\[ \text{Total Initial } \vec{P} = \text{Total Final } \vec{P} \]

If not given information, then:

1. Draw picture in laboratory frame.
2. Subtract cm motion \( \vec{v}_\text{cm} \).
3. Perform the head-on collision in cm.
4. Add cm motion back in.
5. Read off quantities of interest.

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**Elastic Collisions**

8-22 A 10.0-g marble travels to the left with a velocity of magnitude 0.400 m/s on a smooth, level surface and makes a head-on collision with a larger 30.0-g marble moving to the right with a velocity of magnitude 0.200 m/s (Fig. 8-33). If the collision is perfectly elastic, find the velocity of each marble (magnitude and direction) after the collision. (Since the collision is head-on, all the motion is along a line.)

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**Diagram:**

- 10.0 g marble moving to the left at 0.400 m/s
- 30.0 g marble moving to the right at 0.200 m/s
- After collision, the 10.0 g marble moves to the right at 0.100 m/s
- The 30.0 g marble moves to the left at 0.050 m/s

---

**Equations:**

Center of Mass Position:

\[ m_1 x_{cm} = m_1 x_1 + m_2 x_2 \]

Center of Mass Velocity:

\[ m_1 v_{cm} = m_1 v_1 + m_2 v_2 \]

Center of Mass Acceleration:

\[ m_1 a_{cm} = m_1 a_1 + m_2 a_2 \]

**Facts About Collisions:**

1. Center of Mass motion persists when there are no external forces.
2. In a totally INELASTIC collision, relative motion ceases.
3. In a totally ELASTIC collision, momenta reverse in the center-of-mass reference frame.
Consider the collision of a 3 kg block at 5 m/s with a 2 kg block at rest.

**TOTALLY INELASTIC**

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\[ m_{\text{Tot}} v_{\text{cm}} = m_1 v_1 + m_2 v_2 \]
\[ 5 v_{\text{cm}} = 3 \cdot 5 + 2 \cdot 0 \]
\[ v_{\text{cm}} = 3 \]

**SUBTRACT \( v_{\text{cm}} \)**

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**TOTALLY ELASTIC**

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Must use consistent set of units, use gm and cm/s

**before**

\[
\begin{align*}
\text{30} \rightarrow \text{10} \\
\end{align*}
\]

Cons. Mom

\[
\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2
\]

\[
30(20) + 10(-40) = 30 v_{1f} + 10 v_{2f}
\]

**Solve**

\[
v_{2f} = 20 - v_{1f}
\]

\[
2800 = 3 v_{1f}^2 + v_{2f}^2
\]

\[
2800 = 3 v_{1f}^2 + (20 - v_{1f})^2
\]

\[
0 = v_{1f}^2 - 10 v_{1f} - 200
\]

\[
v_{1f} = \frac{10 \pm \sqrt{100 + 4(1)(200)}}{2}
\]

\[
= 20, -10 \text{ cm/s}
\]

Two solutions

\[
\begin{align*}
\text{If } v_{1f} = 20 & \rightarrow v_{2f} = -40 \text{ cm/s} \\
\text{If } v_{1f} = -10 \text{ cm/s} & \rightarrow v_{2f} = 50 \text{ cm/s}
\end{align*}
\]

The first solution is just the before collision case - a trivial soln.