We want to find the distance from the center of the Earth to the center of the Moon. We can use the formula:

\[ F = \frac{GMm}{r^2} \]

where \( F \) is the force, \( G \) is the gravitational constant, \( M \) is the mass of one object, \( m \) is the mass of the other object, and \( r \) is the distance between the centers of the two objects.

First, notice that forces \( F_1 \) and \( F_3 \) will cancel because they have the same magnitude and opposite direction.

\[ F_1 = F_3 = \frac{GM_1 M_3}{r_1^2} \]

Then, we have:

\[ F_2 = F_4 \]

This gives us:

\[ \frac{F_2}{F_3} = \frac{\cos 45^\circ \hat{i} + \cos 45^\circ \hat{j}}{\cos 45^\circ \hat{i} - \cos 45^\circ \hat{j}} \]

\[ \hat{e} = \hat{F}_2 + \hat{F}_3 = 1.18 \times 10^{-11} \hat{i} + 1.18 \times 10^{-11} \hat{j} \]

Now, we can calculate the distance using the formula:

\[ r = \frac{GM}{F} \]

where \( r \) is the distance from the center of the Earth to the center of the Moon, \( G \) is the gravitational constant, \( M \) is the mass of the Earth, and \( F \) is the force.

Finally, we have:

\[ F = \frac{GM_1 M_3}{r^2} \]

where \( F \) is the force, \( G \) is the gravitational constant, \( M_1 \) is the mass of the Earth, \( M_3 \) is the mass of the Moon, and \( r \) is the distance between the centers of the two objects.

So, we can calculate the distance using the formula:

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where \( r \) is the distance from the center of the Earth to the center of the Moon, \( G \) is the gravitational constant, \( M \) is the mass of the Earth, and \( F \) is the force.

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Finally, we have:

\[ F = \frac{GM_1 M_3}{r^2} \]

where \( F \) is the force, \( G \) is the gravitational constant, \( M_1 \) is the mass of the Earth, \( M_3 \) is the mass of the Moon, and \( r \) is the distance between the centers of the two objects.

So, we can calculate the distance using the formula:

\[ r = \frac{GM}{F} \]

where \( r \) is the distance from the center of the Earth to the center of the Moon, \( G \) is the gravitational constant, \( M \) is the mass of the Earth, and \( F \) is the force.

We can use the formula:

\[ r = \frac{GM}{F} \]

where \( r \) is the distance from the center of the Earth to the center of the Moon, \( G \) is the gravitational constant, \( M \) is the mass of the Earth, and \( F \) is the force.

Finally, we have:

\[ F = \frac{GM_1 M_3}{r^2} \]

where \( F \) is the force, \( G \) is the gravitational constant, \( M_1 \) is the mass of the Earth, \( M_3 \) is the mass of the Moon, and \( r \) is the distance between the centers of the two objects.
d) Escape Speed

\[ KE_{e} = 0 \quad U = -G \frac{M}{r} \]

\[ \frac{1}{2} m v_e^2 + \left( -G \frac{M}{r} \right) = 0 \]

\[ v_e = \sqrt{\frac{2GM}{r}} \]

\[ v_e = 1.33 \times 10^3 \text{ m/s} = 1.73 \text{ km/s} \]

b) How high will it go if \( v_e = 1800 \text{ m/s} \)

\[ v_f = \sqrt{\frac{G M}{r}} \]

\[ v_f = 7.5 \times 10^5 \text{ m/s} = 750 \text{ km} \]

or 250 km above surface

13.40 Each escape speed is 11.2 km/s

To determine expressions for escape speed:

\[ \frac{v_e}{R} \]

\[ KE_{e} = \frac{1}{2} m v_e^2 \]

\[ KE_{e} = G \frac{m M}{R} \]

\[ \frac{1}{2} m v_e^2 = G \frac{m M}{R} \]

\[ r_f = \frac{3}{2} R_E \]

\[ KE_{e} = \frac{1}{2} KE_{escape} \]

So

\[ \frac{1}{4} KE_{escape} - G \frac{m M}{R_E} = -G \frac{m M}{R_f^2} \]

\[ \frac{1}{4} G \frac{m M}{R_E} - G \frac{m M}{R_f^2} = -G \frac{m M}{R_f^2} \]

\[ \frac{3}{4} G \frac{m M}{R_E} = -G \frac{m M}{R_f^2} \]

\[ r_f = \frac{3}{2} R_E \]

b) \( KE_i = \frac{1}{2} KE_{escape} \)

So

\[ \frac{1}{2} KE_{escape} - G \frac{m M}{R_E} = -G \frac{m M}{R_f^2} \]

\[ \frac{1}{2} G \frac{m M}{R_E} - G \frac{m M}{R_f^2} = -G \frac{m M}{R_f^2} \]

\[ -\frac{1}{2} G \frac{m M}{R_E} = -G \frac{m M}{R_f^2} \]

\[ r_f = 2R_E \]

c) Tricky wording.

Because Mechanical Energy = KE + Pot'VE

the answer is 2R_E
13.43

Do FBD for the circular motion

\[ m \cdot \frac{v^2}{r} = \sum F_{\text{cent}} \]

\[ m \cdot \frac{v^2}{r} = G \frac{M}{r^2} \]

\[ v^2 = G \frac{M}{r} = \left( \frac{6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \times 5.98 \times 10^{24} \text{ kg}}{2 \times 10^7 \text{ m}} \right) \]

\[ v = 7.80 \times 10^3 \text{ m/s} = 7.8 \text{ km/s} \]

Period:

\[ v = \frac{2 \pi r}{T} \]

\[ 7.80 \times 10^3 = \frac{2 \pi (6.53 \times 10^8)}{T} \]

\[ T = 5.26 \times 10^3 \text{ sec} = 1.46 \text{ hrs} \]

13.45

\[ r = 9.4 \times 10^6 \text{ m} \]

\[ T = 7.39 \text{ hrs} = 2.73 \times 10^4 \text{ sec} \]

We could just start with Kepler's 2-3 Rule

\[ T^2 = \frac{4 \pi^2}{G M} r^3 \]

\[ \left( 2.73 \times 10^4 \right)^2 = \frac{4 \pi^2}{G M} \left( 9.4 \times 10^6 \right)^3 \]

\[ M = 6.38 \times 10^{22} \text{ kg} \]

13.47

\[ r = 2.2 \times 10^7 \text{ m} \]

\[ T = 2.5 \times 10^8 \text{ sec} = 7.85 \times 10^3 \text{ s} \]

\[ T^2 = \frac{4 \pi^2}{G M} r^3 \]

\[ \left( 7.85 \times 10^3 \right)^2 = \frac{4 \pi^2}{G M} \left( 2.2 \times 10^7 \right)^3 \]

\[ M = 1.02 \times 10^{11} \text{ kg} \]

At each star: \( m = 2 \times 10^{33} \text{ kg} \), so there are:

\[ \frac{1.02 \times 10^{11}}{2 \times 10^{33}} \approx 5.1 \times 10^{-22} \text{ stars} \]

13.48

\[ \frac{T^2}{T^2} = \frac{\frac{4 \pi^2}{G M}}{\frac{4 \pi^2}{G M}} \frac{r_M}{r_e} \]

\[ \left( \frac{T_M}{T_e} \right)^2 = \left( \frac{r_M}{r_e} \right)^3 = \left( 1.52 \right)^3 \]

\[ \frac{T_M}{T_e} = 1.87 \]

\[ T_M = 1.87 \times T_e \]

\[ T_M = 1.87 \text{ years} \]

Value in back of book is the same: 1.88 years

13.56

Idea:

\[ \text{Take } r = 10^8 \text{ km} \]

\[ T = 2.3 \times 10^4 \text{ s} \]

\[ a) \quad T^2 = \frac{4 \pi^2}{G M} r^3 \]

\[ \left( 9.72 \times 10^8 \right)^2 = \frac{4 \pi^2}{G M} \left( 100 \times 10^8 \right)^3 \]

\[ M = 6.2 \times 10^{16} \text{ kg} \]

\[ b) \quad V = 14 \times 10^8 \text{ km}^3 = 1.41 \times 10^{13} \text{ m}^3 \]

\[ \text{Density: } \frac{m}{V} = \frac{6.2 \times 10^{16}}{1.41 \times 10^{13}} = 4.44 \times 10^3 \text{ kg/m}^3 \]

\[ 4.44 \text{ g/cm}^3 \]
13.4

\[ M = 5.96 \times 10^4 \text{kg} \]

\[ m = 7.36 \times 10^3 \text{kg} \]

\[ M = 1.09 \times 10^2 \text{kg} \]

\[ r = 3.72 \times 10^8 \text{m} \]

\[ r = 1.5 \times 10^9 \text{m} \]

\[ \frac{F_{\text{moon by Sun}}}{F_{\text{moon by Earth}}} = \frac{G \frac{m \cdot M_s}{r_s^2}}{G \frac{m \cdot M_E}{r_E^2}} \]

\[ = \frac{M_s}{M_E} \left( \frac{r_E}{r_s} \right)^2 \]

\[ = \frac{1.09 \times 10^2}{7.36 \times 10^3} \left( \frac{3.82 \times 10^8}{1.5 \times 10^9} \right)^2 \]

\[ = 2.17 \]

13.5

\[ m_1 = 5 \times 10^{-3} \text{kg} \]

\[ m_2 = 3 \times 10^{-3} \text{kg} \]

\[ m_3 = 1 \times 10^{-3} \text{kg} \]

\[ m_4 = 5 \times 10^{-3} \text{kg} \]

\[ m_5 = 2.5 \times 10^{-3} \text{kg} \]

First, notice that forces \( F_1 \) and \( F_4 \) will cancel because they have the same magnitude and opposite direction.

\[ |F_2| = F_2 = G \frac{m_5 \cdot M_2}{r^2} = (6.67 \times 10^{-11}) \frac{(2.5 \times 10^{-3}) (3 \times 10^{-3})}{(0.1414)^2} \]

\[ = 2.5 \times 10^{-14} \text{ Newtons} \]

\[ |F_3| = F_3 = G \frac{m_5 \cdot M_3}{r^2} = (6.67 \times 10^{-11}) \frac{(2.5 \times 10^{-3}) (1 \times 10^{-3})}{(0.1414)^2} \]

\[ = 8.34 \times 10^{-15} \text{ Newtons} \]

\[ = 0.834 \times 10^{-14} \text{ Newtons} \]

Then

\[ F_2 = F_2 \cos 45^\circ \hat{i} + F_2 \sin 45^\circ \hat{j} \]

\[ F_3 = -F_3 \cos 45^\circ \hat{i} + -F_3 \sin 45^\circ \hat{j} \]

\[ F_{\text{total}} = (F_2 - F_3) \cos 45^\circ \hat{i} + (F_2 - F_3) \sin 45^\circ \hat{j} \]

\[ = 1.18 \times 10^{-14} \hat{i} + 1.18 \times 10^{-14} \hat{j} \text{ Newtons} \]
13.27

\[ F = \frac{G M_{\text{sun}} m}{r^2} \]

We want to find \( F \) for Earth.

- \( M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg} \)
- \( m = 5.97 \times 10^{24} \text{ kg} \)
- \( r = 1.5 \times 10^7 \text{ m} \)

\[ F = \frac{G (1.99 \times 10^{30}) (5.97 \times 10^{24})}{(1.5 \times 10^7)^2} \]

\[ F = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 5.97 \times 10^{24}}{2.25 \times 10^{14}} \]

\[ F = 3.92 \times 10^{-6} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \]

A) \( g \) at surface of earth

\[ g = \frac{F}{m} \]

\[ g = \frac{3.92 \times 10^{-6} \times 1.8 \times 10^{17}}{5.97 \times 10^{24}} \]

\[ g = 9.76 \text{ m/s}^2 \]

B) \( g \) at top of mantle

\[ g = \frac{M_{\text{mantle}}}{r^2} \]

\[ g = \frac{(6.67 \times 10^{-11} \times 4.1 \times 10^{12})}{(3.94 \times 10^7)^2} \]

\[ g = \frac{2.77 \times 10^{-2}}{3.32 \times 10^{15}} \]

\[ g = 8.83 \times 10^{-17} \text{ m/s}^2 \]

C) Suppose Earth were uniform density. Calc. \( g \)

\[ M = \frac{4}{3} \pi R^3 \rho \]

\[ M = \frac{4}{3} \pi (6.4 \times 10^6)^3 \times 5.5 \times 10^3 \]

\[ M = 5.98 \times 10^{24} \text{ kg} \]

\[ \rho = \frac{M}{4/3 \pi R^3} \]

\[ \rho = \frac{5.98 \times 10^{24}}{4/3 \pi (6.4 \times 10^6)^3} \]

\[ \rho = 5.5 \times 10^3 \text{ kg/m}^3 \]

\[ g = \frac{GM}{r^2} \]

\[ g = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.4 \times 10^6)^2} \]

\[ g = 9.82 \text{ m/s}^2 \]

So

\[ g = \frac{GM}{r^2} \]

\[ g = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{(6.4 \times 10^6)^2} \]

\[ g = 9.75 \text{ m/s}^2 \]
13.39

A) What speed will it hit surface if dropped from 1600 km above surface?

\[ KE = 0 \quad U = -\frac{G \cdot m \cdot M}{r} \]

\[ v_f = \sqrt{\frac{2 \cdot G \cdot M}{r}} = \sqrt{\frac{2 \cdot (6.67 \times 10^{-11}) \cdot (1.124 \times 10^{22})}{500 \times 10^3}} \]

\[ v_f = 1.73 \times 10^3 \text{ m/s} = 1.73 \text{ km/s} \]

b) How high will it go if \( v_e = 1860 \text{ m/s} \)

\[ KE = 0 \quad U = -\frac{G \cdot m \cdot M}{r_f} \]

\[ \frac{1}{2} m v_e^2 + -\frac{G \cdot m \cdot M}{r} = 0 + -\frac{G \cdot m \cdot M}{r_f} \]

\[ \frac{1}{2} (1860)^2 - (6.67 \times 10^{-11}) \frac{1.124 \times 10^{22}}{500 \times 10^3} = - (6.67 \times 10^{-11}) \frac{1.124 \times 10^{22}}{r_f} \]

\[ r_f = 7.5 \times 10^5 \text{ m} = 750 \text{ km} \]

or 250 km above surface
Earth escape speed = 11.2 km/s

To determine expression for escape speed:

\[ \frac{v_e}{R_E} \]

Total Initial Energy = Total Final Energy

\[ \frac{1}{2}mv_e^2 + -G \frac{mM}{R_E} = 0 + 0 \]

\[ KE_{\text{escape}} = -G \frac{mM}{R_E} \]

\[ \frac{1}{2}mv_e^2 = G \frac{mM}{R_E} \]

Now after we've got these formulae, set the actual problem up.

\[ KE = 0 \]

\[ KE_{\text{escape}} = -G \frac{mM}{R_E} \]

\[ \frac{1}{2}mv_e^2 = G \frac{mM}{r_f} \]

a) \[ v_e = \frac{1}{2} \text{ escape speed} \]

Note that this implies \( KE_e = \frac{1}{4} KE_{\text{escape}} \)

So

\[ \frac{1}{4} KE_{\text{escape}} - G \frac{mM}{R_E} = -G \frac{mM}{r_f} \]

\[ \frac{1}{4} G \frac{mM}{R_E} - G \frac{mM}{R_E} = -G \frac{mM}{r_f} \]

\[ -G \frac{mM}{R_E} = -G \frac{mM}{r_f} \]

\[ r_f = \frac{4}{3} R_E \]

b) \[ KE_e = \frac{1}{2} KE_{\text{escape}} \]

So

\[ \frac{1}{2} KE_{\text{escape}} - G \frac{mM}{R_E} = -G \frac{mM}{r_f} \]

\[ \frac{1}{2} G \frac{mM}{R_E} - G \frac{mM}{R_E} = -G \frac{mM}{r_f} \]

\[ -G \frac{mM}{R_E} = -G \frac{mM}{r_f} \]

\[ r_f = 2R_E \]

c) tricky wording.

Because Mechanical Energy = KE + Potential Energy

The answer is Zero.
Do fbd for the circular motion
\[ m\cdot \frac{v^2}{r} = \frac{GM^2}{r^2} \]
\[ v^2 = \frac{GM}{r} = \frac{6.67 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2} \times 5.96 \times 10^{24} \text{ kg}}{6370 \text{ km} + 160 \text{ km}} = \frac{5.96 \times 10^{24}}{6530 \times 10^3} \]
\[ V = 7.80 \times 10^3 \text{ m/s} = 7.8 \text{ km/s} \]

Revised
\[ V = \frac{2\pi r}{T} \]
\[ 7.80 \times 10^3 = \frac{2\pi (6530 \times 10^3)}{T} \]
\[ T = 5.26 \times 10^3 \text{ sec} = 1.46 \text{ hrs} \]

13.43

We could just start with Kepler's 2-3 Rule
\[ T^2 = \frac{4\pi^2}{GM} r^3 \]
\[ (2.754 \times 10^6)^2 = \frac{4\pi^2}{(6.67 \times 10^{-11})} \times (9.4 \times 10^6)^3 \]
\[ M = 6.48 \times 10^{23} \text{ kg} \]

13.45

\[ r = 9.4 \times 10^6 \text{ m} \]
\[ T = 7h 39m = 2.754 \times 10^4 \text{ sec} \]

\[ T^2 = \frac{4\pi^2}{GM} r^3 \]
\[ (7.85 \times 10^{15})^2 = \frac{4\pi^2}{(6.67 \times 10^{-11})} \times (2.2 \times 10^{20})^3 \]
\[ M = 1.02 \times 10^{41} \text{ kg} \]

If each star is \( \sim 2 \times 10^{36} \text{ kg} \), then there are
\[ \frac{1.02 \times 10^{41}}{2 \times 10^{36}} \sim 5.1 \times 10^5 \text{ stars} \]
13.48

\[ T^2 = \frac{4\pi^2}{GM} r^3 \]

Form a ratio

\[ \frac{T_m^2}{T_e^2} = \frac{\frac{4\pi^2}{GM_m} r_m^3}{\frac{4\pi^2}{GM_e} r_e^3} \]

\[ \left( \frac{T_m}{T_e} \right)^2 = \left( \frac{r_m}{r_e} \right)^3 = \left(1.52 \right)^3 \]

\[ \frac{T_m}{T_e} = 1.87 \]

\[ T_m = 1.87 \, T_e \]

\[ T_m = 1.87 \, \text{years} \]

Value in back of book is the same ~ 1.88 years

13.56

Ida, Dactyl take \( r \approx 100 \, \text{km} \)

\[ T = 27h = 9.72 \times 10^4 \, \text{s} \]

a) Use

\[ T^2 = \frac{4\pi^2}{GM} r^3 \]

\[ \left( 9.72 \times 10^4 \right)^2 = \frac{4\pi^2}{(6.67 \times 10^{-11}) M} \left( 100 \times 10^3 \right)^3 \]

\[ M = 6.26 \times 10^{24} \, \text{kg} \]

b) Volume

\[ 14100 \, (\text{km}^3) = 1.41 \times 10^{13} \, \text{m}^3 \]

\[ \text{density} = \frac{\text{mass}}{\text{vol}} = \frac{6.26 \times 10^{24}}{1.41 \times 10^{13}} = 4.44 \times 10^3 \, \text{kg/m}^3 \]

\[ = 4.44 \, \text{g/cm}^3 \]