11.24  

a) Earth  
\[ M = 5.96 \times 10^{24} \text{ kg} \]
\[ R = 6.370 \text{ km} = 6.370 \times 10^6 \text{ m} \]
\[ \omega = \frac{2\pi}{T} = \frac{2\pi}{24 \text{ hrs}} = \frac{2\pi}{86400 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s} \]

Solid ball  
\[ I = \frac{2}{5} MR^2 \]
\[ L = I\omega = \frac{2}{5} (5.96 \times 10^{24}) (6.370 \times 10^6)^2 \left( 7.27 \times 10^{-5} \text{ rad/s} \right) \]
\[ = 4.48 \times 10^{40} \text{ kg m}^2/\text{s} \]

b) The Earth is effectively a point object. We could use the formula \( L = I\omega \) or \( L = mr^2\omega \). I think I'll use \( I\omega \) because it's perhaps slightly quicker.

\[ I = mr^2 = \frac{2\pi}{3} \text{ (Note)} \]
\[ m = 5.96 \times 10^{24} \text{ kg} \]
\[ r = 1.5 \times 10^8 \text{ m (Note)} \]

\[ L = I\omega = mr^2\omega = \left( 5.96 \times 10^{24} \right) (1.5 \times 10^8)^2 \left( 2.7 \times 10^{-7} \right) \]
\[ = 2.68 \times 10^{40} \text{ kg m}^2/\text{s} \]

c) Interestingly, they are roughly the same size.
The people walking on the ring should have the same sensation as walking on the Earth. This means that the normal force they experience should be the same as if they were standing on Earth.

\[
M_{newton} = \frac{2}{R} \text{ Fnd}
\]

\[
m \frac{v^2}{R} = N_{space \ matter}
\]

we now \[ N_{space} = N_{Earth} = mg \]

\[
M \frac{v^2}{R} = mg
\]

\[
v^2 = gR \rightarrow v = \sqrt{(9.8)(100m)} = 31.3 \text{ m/s}
\]

The angular speed would be

\[
v = R \omega
\]

\[
31.3 = 100 \omega
\]

\[
\omega = 0.313 \text{ rad/s}
\]

Angular Momentum

\[
L = I \omega = M R^2 \omega = (5 \times 10^4 \text{ kg})[100]^2 \times 0.313
\]

\[
\text{this cylinder} = 1.565 \times 10^8 \text{ kg m}^2 \text{ s}
\]

b) How long to fire rockets?

\[
\alpha = 2 \text{ F}
\]

\[
\alpha = 2 \text{ RF}
\]

\[
MR^2 \alpha = 2 \text{ RF}
\]

\[
(5 \times 10^4 \text{ N})(100) \alpha = 2 \text{ (100)} (150)
\]

\[
\alpha = 6.0 \times 10^{-5} \text{ rad/s}
\]

Then

\[
\omega = \omega_0 + \alpha t
\]

\[
0.313 = (6 \times 10^{-5}) t
\]

\[
t = 5.22 \times 10^3 \text{ sec}
\]

c) Work

\[
\frac{dL}{dt} = 2 \text{ F}
\]

\[
\frac{\Delta L}{\Delta t} = 2 F
\]

\[
\Delta L = (2 F) \Delta t
\]

\[
1.565 \times 10^8 = \left[2 \times 100 \times 150\right] (5.22 \times 10^3)
\]

\[
? = 1.565 \times 10^8
\]
11.32

\[ \omega_i = 0.75 \text{ rad/s} \]

\[ \omega_f = \frac{5}{3} \]

\[ \text{a)} \]

\[ \text{Init Arg Mom} = \text{Final Arg Mom} \]

\[ \text{Init} \ \omega_{\text{init}} = \text{Final} \ \omega_{\text{final}} \]

\[ \omega_{\text{final}} = \frac{0.5}{0.5} \]

\[ \frac{0.5}{0.5} \]

\[ \text{I}_{\text{init}} = \text{I}_{\text{incident}} + \text{I}_{\text{dumbbell}} + \text{I}_{\text{ball}} \]

\[ \text{I}_{\text{final}} = \text{I}_{\text{incident}} + \text{I}_{\text{dumbbell}} + \text{I}_{\text{ball}} \]

\[ \begin{bmatrix} \text{I}_{\text{init}} + 2 \text{I}_{\text{dumbbell}} \end{bmatrix} \omega_{\text{init}} = \begin{bmatrix} \text{I}_{\text{final}} + 2 \text{I}_{\text{dumbbell}} \end{bmatrix} \omega_{\text{final}} \]

\[ \begin{bmatrix} \text{I}_{\text{init}} + 2 \text{M}_c^2 \end{bmatrix} \omega_{\text{init}} = \begin{bmatrix} \text{I}_{\text{final}} + 2 \text{M}_f^2 \end{bmatrix} \omega_{\text{final}} \]

\[ \frac{3 + 2(3)(1)^2}{9} \Omega_{\text{final}} = \frac{3 + 2(3)(0.5)^2}{3.5^2} \Omega_{\text{final}} \]

\[ \Omega_{\text{final}} = 1.91 \text{ rad/sec} \]

\[ \text{b)} \text{KE Kinetic Energies} \]

\[ \text{KE}_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2} (9)(0.75)^2 = 2.53 \text{ Joules} \]

\[ \text{KE}_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2} (3.5^2)(141)^2 = 6.46 \text{ Joules} \]

11.37

Collision occurs so use conservation of momentum. In this case angular momentum, since rotators are involved.

\[ \text{Init} \ \text{L} = \text{Final} \ \text{L} \]

\[ \text{calculate the initial L using} \]

\[ \text{the impact parameter} \]

\[ l \text{MV}_0 = I \omega_{\text{recoil}} \]

\[ l \text{MV}_0 = (m + M) \frac{l^2}{2} \omega_{\text{recoil}} \]

\[ \omega = \frac{m}{m + M} \frac{1}{l} \text{V}_0 \]

\[ \text{Initial KE} \]

\[ \text{KE}_i = \frac{1}{2} m \text{V}_0^2 \]

\[ \text{Final KE} \]

\[ \text{KE}_f = \frac{1}{2} I \omega^2 = \frac{1}{2} \left( \frac{m + M}{m} \right)^2 \left( \frac{m}{m + M} \frac{1}{l} \text{V}_0 \right)^2 \]

\[ = \frac{1}{2} m \text{V}_0^2 \]

\[ = \frac{1}{2} m \text{M} \text{V}_0^2 \]

\[ \text{Can write} \]

\[ \text{KE}_f = \frac{m}{m + M} \text{KE}_i \]
11.46

a) Speed at B
\[ mgh = \frac{1}{2} m v_B^2 \]
\[ 9.8 \times 6.3 = \frac{1}{2} v_B^2 \]
\[ v_B = 11.01 \text{ m/s} \]

b) Angular momentum about center of curvature while at B
\[ L = r p = r m v \]
\[ = 6.3 \times (11.1) \times (11.1) \]
\[ = 5.31 \times 10^3 \text{ kg m}^2/\text{s} \]

c) Angular momentum is constant because no Torque.

The forces are parallel and perpendicular to \( \vec{r} \)

d) Initial \( L = \) Final \( L \)
\[ v_i p_i = v_f p_f \]
\[ v_i m v_i = v_f m v_f \]
\[ 6.3 \times (11.1) = 5.85 \times v_f \]
\[ v_f = 11.95 \text{ m/s} \]

\[ m = 76 \text{ kg} \]

e) Change in kinetic energy
\[ KE_i = \frac{1}{2} m v^2 = \frac{1}{2} 76 (11.1)^2 = 4.68 \times 10^3 \text{ J} \]
\[ KE_f = \frac{1}{2} m v^2 = \frac{1}{2} 76 (11.95)^2 = 5.43 \times 10^3 \text{ J} \]
\[ \Delta KE = 0.75 \times 10^3 = 750 \text{ J} \]

f) Speed at D
\[ \text{Tot Energy} D = \text{Total Energy} \]
\[ \frac{1}{2} m v_D^2 = \frac{1}{2} m v_B^2 + mgh \]
\[ \frac{1}{2} (11.95)^2 = \frac{1}{2} v_D^2 + 9.8 \times 9.85 \]
\[ v_D = 5.30 \text{ m/s} \]

g) How high?
\[ h = \frac{-F V_b}{9.30} \]
\[ \text{Pott energy at top of 1/2 pipe} \]
\[ \frac{1}{2} m v_D^2 = mgh \]
\[ \frac{1}{2} (5.3)^2 = 9.8 \times h \]
\[ h = 1.44 \text{ m} \]

b) Next page.
11.46 cont

h) How long air borne?

\[ y(t) = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 \]

\[ y(0) = 0 \]

\[ v_{0y} = 5.30 \text{ m/s} \]

\[ a_y = -9.8 \text{ m/s}^2 \]

\[ y(t) = 5.30 t + \frac{1}{2} (-9.8) t^2 \]

\[ -2.34 = 5.30 t + \frac{1}{2} (-9.8) t^2 \]

\[ 4.9t^2 - 5.30t - 2.34 = 0 \]

\[ t = \frac{5.30 \pm \sqrt{(5.30)^2 - 4(4.9)(-2.34)}}{2(4.9)} \]

\[ t = \frac{5.30 \pm 3.40}{9.8} = \approx 1.42 \text{ sec} \]

\[ t = 1.42 \text{ sec} \]

This problem just goes on forever.
13.15 Io, satellite of Jupiter

\[ T^2 = \frac{4\pi^2}{GM} r^3 \]

- \( T = 1.77 \text{ d} \)
  - \( = 1.53 \times 10^5 \text{ s} \)
- \( r = 4.22 \times 10^8 \text{ km} \)
  - \( = 4.22 \times 10^5 \text{ m} \)
- \( G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \)
- \( M = 1.90 \times 10^{27} \text{ kg} \)

\[ (1.53 \times 10^8)^2 = \frac{4\pi^2}{GM} (4.22 \times 10^8)^3 \]

13.17 "Geostationary"
Synchronous Satellite above Earth.

\[ T^2 = \frac{4\pi^2}{GM} r^3 \]

\[ (86400)^2 = \frac{4\pi^2}{GM} r^3 \]

- \( T = 1 \text{ day} \)
  - \( = 86400 \text{ sec} \)
- \( r = 4.221 \times 10^7 \text{ m} \)
  - \( = 4.221 \times 10^4 \text{ km} \)
- \( M = 5.96 \times 10^{24} \text{ kg} \)
  - \( = 42240 \text{ km} \)

\( \frac{42240}{60370} = 35840 \text{ km} \)

This orbit tends to be used for communications satellites.
Geosynchronous orbit

From Wikipedia, the free encyclopedia

A geosynchronous orbit is an orbit around the Earth with an orbital period matching the Earth's sidereal rotation period\(^1\). This synchronization means that for an observer at a fixed location on Earth, a satellite in a geosynchronous orbit returns to exactly the same place in the sky at exactly the same time each day. In principle, any orbit with a period equal to the Earth's rotational period is technically geosynchronous; however, the term is often used\(^2\) to refer to the special case of a geosynchronous orbit that is circular (or nearly circular) and at zero (or nearly zero) inclination, that is, directly above the equator. This is customarily called a geostationary orbit.

A semisynchronous orbit has an orbital period of 0.5 sidereal days, i.e. 11 h 58 min. Relative to the Earth's surface it has twice this period, and hence appears to go around the Earth once every day. Examples include the Molniya orbit and the orbits of the satellites in the Global Positioning System.

### Contents

- 1 Orbital characteristics
- 2 Geostationary orbit
- 3 Synchronous orbits around general astronomical objects
- 4 Other geosynchronous orbits
- 5 History
- 6 See also
- 7 References
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### Orbital characteristics

All geosynchronous orbits have a semi-major axis of 42,164 km (26,199 mi).\(^3\) In fact, orbits with the same period share the same semi-major axis: \(a = \sqrt[3]{\frac{\mu}{P^2}}\) where \(a=\)semi-major axis, \(P=\)orbital period, \(\mu=\)geocentric gravitational constant.

In the special case of a geostationary orbit, the ground track of a satellite is a single point on the equator. In the general case of a geosynchronous orbit with a non-zero inclination or eccentricity, the ground track is a more or less distorted figure-eight, returning to the same places once per solar day.

### Geostationary orbit

*Main article: Geostationary orbit*
Geostationary orbit

From Wikipedia, the free encyclopedia

A geostationary orbit (or Geostationary Earth Orbit - GEO) is a geosynchronous orbit directly above the Earth's equator (0° latitude), with a period equal to the Earth's rotational period and an orbital eccentricity of approximately zero. These characteristics are required so that, from locations on the surface of the Earth, geostationary objects appear motionless in the sky, making the GEO an orbit of great interest to operators of communications and weather satellites. Due to the constant 0° latitude and circularity of geostationary orbits, satellites in GEO differ in location by longitude only.

The notion of a geosynchronous satellite for communication purposes was first published in 1928 (but not widely so) by Herman Potočnik.[1] The idea of a geostationary orbit was first published on a wide scale in a paper entitled "Extra-Terrestrial Relays — Can Rocket Stations Give Worldwide Radio Coverage?" by Arthur C. Clarke, published in Wireless World magazine in 1945.[2] In this paper, Clarke described it as a useful orbit for communications satellites. As a result this is sometimes referred to as the Clarke Orbit.[3] Similarly, the Clarke Belt is the part of space approximately 36,000 km (22,000 mi) above sea level, in the plane of the equator, where near-geostationary orbits may be implemented. The Clarke Orbit is about 265,000 km (165,000 mi) long.

Geostationary orbits are useful because they cause a satellite to appear stationary with respect to a fixed point on the rotating Earth. As a result, an antenna can point in a fixed direction and maintain a link with the satellite. The satellite orbits in the direction of the Earth's rotation, at an altitude of 35,786 km (22,236 mi) above ground. This altitude is significant because it produces an orbital period equal to the Earth's period of rotation, known as the sidereal day.

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- 1 Introduction
- 2 Derivation of geostationary altitude
- 3 Practical limitations
  - 3.1 Communications
- 4 Orbit allocation
- 5 See also
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- 7 External links

Introduction
13.21 Each mass \( M \) causes a gravitational force on mass "m".

Each mass \( M \) causes a gravitational field on mass "m".

Since the force and the field are vectors, they must be added as such.

Recall \( |\vec{F}_{grav}| = G \frac{Mm}{r^2} = m \left( \frac{GM}{r^2} \right) = mg \)

so that \( \vec{g} = \frac{GM}{r^2} \)

\#1 \( |\vec{g}_1| = \frac{GM}{r^2} \) and \( \vec{g}_1 = 0 \hat{\imath} + \frac{GM}{r^2} \hat{j} \)

\#2 \( |\vec{g}_2| = \frac{GM}{(2r)^2} \) and \( \vec{g}_2 = \frac{GM}{2r^2} \hat{\imath} + 0 \hat{j} \)

\#2 \( |\vec{g}_2| = \frac{GM}{(2r)^2} \) and \( \vec{g}_2 = \frac{GM}{2r^2} \hat{\imath} + 0 \hat{j} \)

\( \cos 45^\circ = \frac{\sqrt{2}}{2} \quad \sin 45^\circ = \frac{\sqrt{2}}{2} \)

Then \( \vec{g}_1 + \vec{g}_2 + \vec{g}_3 = \left( \frac{GM}{r^2} + \frac{\sqrt{2} GM}{2r^2} \right) \hat{\imath} + \left( \frac{GM}{r^2} + \frac{\sqrt{2} GM}{2r^2} \right) \hat{j} \)

\[ = \frac{GM}{r^2} \left[ \left( 1 + \frac{\sqrt{2}}{4} \right) \hat{\imath} + \left( 1 + \frac{\sqrt{2}}{4} \right) \hat{j} \right] \]

13.26

Well of course I would ignore air resistance... \( \frac{v_i}{v_f} = \frac{4}{5} \)

\( v_i: 10 \text{ km/s} \)

\( r_i: 6370 \text{ km} \)

\( r_f: 6370 \times 10^6 \text{ km} \)

\( \frac{1}{2} m v_i^2 + -G \frac{mM}{r_i} = 0 + -G \frac{mM}{r_f} \)

\( \frac{1}{2} (10^5)^2 - (6.67 \times 10^{-11}) \frac{(5.86 \times 10^7)}{6370} = \frac{(6.67 \times 11)}{6370} \frac{(5.86 + 24)}{r_f} \)

\( r_f = 3.204 \times 10^7 \text{ m} \)

\( = 3.204 \times 10^4 \text{ km} \)

\( = 32040 \text{ km} \)

Altitude \( 32040 - \frac{6370}{25670} \text{ km} \)
13.35

$w = 200 \text{ kg}$

$\text{altitude} = 200 \text{ km}$

$r = (6370 + 200) = 6570 \text{ km}
= 6.570 \times 10^6 \text{ m}$

a) $T^2 = \frac{4\pi^2}{GM} r^3$

$T^2 = \frac{4\pi^2}{(6.67 \times 10^{-11})(5.96 \times 10^2)} (6.570 \times 10^6)^3$

$T = 5304 \text{ sec}$

b) Speed

$V = \frac{2\pi r}{T} = \frac{2\pi (6.570 \times 10^6)}{5304}$

$= 7779 \times 10^3 \text{ m/s}$

$= 7779 \text{ m/s}$

c) Total Energy of Satellite in orbit

$\frac{1}{2}mv^2 + \frac{GMm}{r}$

$\frac{1}{2} (200)(7779)^2 - \frac{(200)(5.96 \times 10^2)}{(6.570 \times 10^6)}$

$-6.050 \times 10^8 \text{ Joules}$