

Manipulation of Large Objects by Swarms of Autonomous Marine Vehicles: Part I - Rotation

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Abstract—In this paper, the control problem of utilizing a team of autonomous marine vehicles (i.e., tug boats) cooperating to manipulate a larger floating object (i.e., disabled ship) while operating in a decentralized architecture is considered. After decomposing the problem into several phases, a control design targeting the issue of inducing controlled rotations for the manipulated object is presented.

I. Introduction

Swarms - large groups of relatively simple and cheap robots working in concert without a centralized decision maker - have been a popular area of research over the past few years. Land-based mobile robots are the most typical type of swarm considered although some work on unmanned aerial vehicles (UAVs) have emerged in recent years. Most of the current literature considers using the distributed sensing capabilities of the group for reconnaissance and information gathering. In contrast, few works actually address the issue of how swarms can influence their surrounding environment.

The goal of this project is to allow a single human operator exert high-level control of the on-water manipulation of large objects using swarms of autonomous marine (surface) vessels. On-water manipulation of large objects by a swarm of small autonomous vehicles is a novel area of investigation with naval applications to marine ordinance disposal, transportation of disabled ships, and assembly of large marine structures such as positioning sonar arrays for littoral surveillance or construction of off-shore platforms and bases. Aside from these direct applications, the methodology developed will have indirect application to land-based robot swarms, swarm reconnaissance and exploration missions, and micro-assembly tasks.

Distributed manipulation is a topic considered in the robotics literature, although it is generally limited to kinematic analysis of small numbers of robots with centralized decision making. In addition, the authors are not aware of any work considering the unique dynamics of marine vehicles. A rigorous second order dynamic analysis of controllability and manipulability issues related to pushing objects with single robot is presented in [6].

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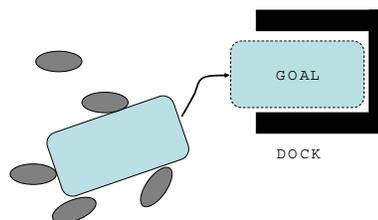


Fig. 1. The small gray ovals are autonomous marine vehicles which manipulate the unactuated larger green floating object (e.g., disabled ship) to the goal position.

Dynamically manipulating objects using two or three robots was examined in [8] and [7]. In both cases it is unclear how to extend the methodologies to many robots with decentralized decision making. Perhaps the most relevant literature to this proposal is the work on “caging” - using a relaxed version of form closure for cooperative manipulation - presented in [9], [10] and [11]. Controllers are designed which force robots to surround the object. Inter-robot spacing is constrained to be small enough that it is impossible for it to “escape”, meaning that as the robots move, so must the object. The primary drawback of both these approaches is that they are strictly kinematic analysis. The extension to on water manipulation requires consideration of hydro-dynamic forces, non-negligible drift, and disturbance rejection.

The outline of this paper is as follows. After a discussion of related work below, an overview of the problem is given in Section II. The problem is too rich to be considered in its entirety here, so we choose to focus on the sub-problem of controlling the orientation of the manipulated object in this work. A formal problem statement appears at the end of Section V. A robust, decentralized control strategy for each swarm member is presented in Section VI. The stability analysis for the developed control design is presented in VII. The conclusion and future work are presented in VIII

II. Overview

The problem we are ultimately concern with is shown in Figure 1. Given a large, unactuated floating object with position and yaw-orientation $\eta(t)$, and a desired position and orientation, $\eta_d(t)$, utilize N small autonomous marine vehicles (swarm members) to affect the motion. The general problem can be decomposed into the following subtasks.

- 1) Establish Contact: The location of the initial position of the members is arbitrary so they cannot manipulate the object before making contact with it. Robots must select and move to some point on the perimeter of the object.
- 2) Rotate: Large objects such as ships have a preferential translation axis due to drag and added mass terms. The robots must rotate the object until this preferred moment axis is aligned with $(q_d - q)$.
- 3) Translate: Execute a pure translation of the object from the current position q to the destination position q_d .
- 4) Rotate: Rotate object from current orientation ψ to the destination orientation ψ_d .

The topic of obstacle avoidance is not considered within the scope of this paper. Rather, we will consider issues associated with Phases 1, 2, and 4.

III. Contact Considerations

In the typical dynamic positioning or dynamic tracking problem, the motion of a given vessel is to be controlled by a given set of installed actuators (propellers, rudders, tunnel thrusters, azimuth thrusters, etc.) The dynamic characteristics of the vessel are assumed to be known. Actuator locations are usually fixed by vessel design, and actuator forces are well modeled; hence the control effects of actuators on vessel surge, sway, and yaw motions are consistent and fairly predictable. The solution of this problem involves finding the set of control inputs, generally the set of actuator force magnitudes, directions, and sequences to achieve a desired vessel motion.

Our work involves the more general problem wherein actuators are not at fixed locations on the vessel to be positioned. In our work the actuators are separate vessels (tugs), which exert forces on the vessel to be moved. This scenario brings additional degrees of freedom to the problem through the ability to position the actuators along the ship's waterline. These additional degrees of freedom allow for further optimization of control forces beyond that of the typical problem.

The force interface between the tug and vessel imposes constraints on actuator forces, as actuator force directions may be constrained by achievable tug orientations or by differences in forward/reverse thrust capabilities. When the tug is not connected to the vessel, the only allowable actuation force is a combination of pushing and friction forces between the tug and vessel. Slipping between the two is undesirable in that it changes actuator location. If slipping is to be avoided, the tug thrust component parallel to the vessel's waterline must not exceed the static friction coefficient, or

$$\sin \theta \leq \mu_s \cos \theta \quad (1)$$

where θ is the angle between the thrust vector and the normal to the vessel's waterline and μ_s is the static

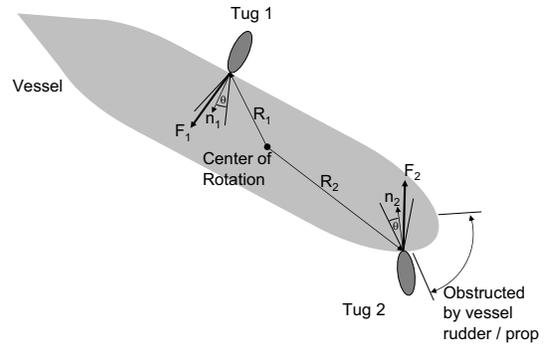


Fig. 2. Illustration of swarm forces on disabled vessel.

coefficient of friction. The maximum allowable value of θ is therefore given by the following expression

$$\theta_{\max} = \tan^{-1}(\mu_s) \quad (2)$$

In such cases, force direction is limited to the normal to the surface and the range of angles near the normal for which friction is sufficient to prevent the tug from slipping significantly. A likely scenario is illustrated in Figure 2, for two tugs not connected to the vessel while controlling vessel yaw. Force directions are limited to the normal vector to the waterline $\vec{n}_i \pm \theta$. Tugs may be positioned along the vessel waterline except where vessel structures prohibit pushing, such as the propeller/rudder area. The net torque applied to the vessel is

$$\vec{\tau}_{\text{net}} = \sum_{i=1}^N \vec{R}_i \times \vec{F}_i \quad (3)$$

Optimization of tug location to maximize available torque to control yaw involves maximizing (3), subject to hull geometry and allowable force directions. A similar optimization process for the net force

$$\vec{F}_{\text{net}} = \sum_{i=1}^N \vec{F}_i \quad (4)$$

is possible for surge and sway motions. Overall surge/sway/yaw control can be optimized by simultaneous force and torque optimization subject to the above constraints. When the tug is connected to the vessel with lines, achievable actuation forces are no longer limited by friction. However tug locations will be limited to the available connection points installed on the vessel. Additionally, although a wider range of actuation force directions is available, changing direction may involve significant time delays due to the time required to change the relative orientation of the tug and vessel.

Our generalization also brings additional uncertainty. Whereas the focus of the typical problem is the vessel, whose characteristics are well known; this work involves the positioning of an arbitrary vessel whose dimensions, mass, hydrodynamic coefficients, and other characteristics will be uncertain or unknown. As a result, our

work is centered on a group of tugs whose characteristics are known and which represent the majority of known characteristics in the problem. The identity of the vessel to be moved may not be known until the tugs are assigned to move it.

IV. Contact Model

Once contact is established with the disabled vessel, the swarm vehicles appear in essence as independent azimuth thrusters; therefore, the three degree of freedom, kinematic/dynamic equation of motion for a disabled vessel operating in a body-fixed reference frame and actuated through N autonomous vehicles is governed by the following expression [3]

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= R(\psi) \mathbf{v} \\ M\dot{\mathbf{v}} + C(\mathbf{v})\mathbf{v} + D(\mathbf{v})\mathbf{v} &= \mathbf{B}\mathbf{u} \end{aligned} \quad (5)$$

where the vector $\boldsymbol{\eta} = [x \ y \ \psi]^T \in \mathbb{R}^3$ denotes the inertial frame position and rotation, $\mathbf{v} = [u \ v \ r]^T \in \mathbb{R}^3$ represents the body fixed translational and yaw velocities, the rotation matrix $R(\psi) \in \mathbb{R}^{3 \times 3}$ that relates body fixed coordinate system to an inertial coordinate system is given by the following matrix

$$R(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$M \in \mathbb{R}^{3 \times 3}$ represents the system inertia matrix (including added mass), $C(\mathbf{v}) \in \mathbb{R}^{3 \times 3}$ denotes the Coriolis-centripetal matrix (including added mass), $D(\mathbf{v}) \in \mathbb{R}^{3 \times 3}$ captures damping effects, $\mathbf{B} \in \mathbb{R}^{3 \times N}$ denotes the swarm configuration matrix whose i^{th} column is given by the following

$$\mathbf{B}_i = \begin{bmatrix} \sin \alpha_i \\ \cos \alpha_i \\ -l_{iy} \cos \alpha_i + l_{ix} \sin \alpha_i \end{bmatrix} \quad 1 < i \leq N \quad (7)$$

where $l_{ix}, l_{iy} \in \mathbb{R}^1$ represent the distance from the disabled vehicle's center of mass to the i^{th} swarm vehicle contact point, $\alpha_i \in \mathbb{R}^1$ denotes angle at which the i^{th} swarm vehicle force is applied, and $\mathbf{u} \in \mathbb{R}^{N \times 1}$ is the swarm vehicle input thrust vector.

V. Problem Formulation

In this phase of the project, the objective is to develop a yaw ($\psi(t) \in \mathbb{R}^1$) tracking control scheme for a disabled vessel actuated through N autonomous vehicles spaced at fixed points surrounding the vessel (constant \mathbf{B} matrix). It will be assumed that the autonomous vehicles are securely attached to the hull of the disabled vessel in a manner so as to provide both forward and reverse thrust directions. In addition, a decentralized architecture within the swarm is assumed; therefore, each swarm vehicle will only be aware of the disabled vessel's position and orientation as well as its relative location from the disabled vessel's center of mass (i.e., l_{ix} and l_{iy}). Our objective here is to design a generic swarm vehicle

thrust input $u_i(t)$ such that $\psi(t)$ tracks a sufficiently smooth desired orientation trajectory $\psi_d(t) \in \mathbb{R}^1$ (i.e., $\psi_d(t), \dot{\psi}_d(t), \ddot{\psi}_d(t), \in L_\infty$). To this end, the orientation tracking error signal $e_\psi(t) \in \mathbb{R}^1$ is defined in the following manner

$$\begin{aligned} e_\psi &= [0 \ 0 \ 1] \mathbf{e}_\eta \\ \mathbf{e}_\eta &= \dot{R}^T(\psi) (\boldsymbol{\eta} - \boldsymbol{\eta}_d) \end{aligned} \quad (8)$$

where $\mathbf{e}_\eta \in \mathbb{R}^3$ represents the 3-DOF tracking error signal. In lieu of separating out the yaw dynamics from the translational dynamics of (5), the orientation tracking controller will be developed in terms of the 3-DOF tracking problem with the orientation tracking control input being extracted from from the 3-DOF control input vector. In addition, the swarm thruster control design will assume exact model knowledge of the disabled vessel's parameters (i.e., M , $C(\mathbf{v})$, and $D(v)$) as well as full state feedback (i.e., the translational position and rotational vector $\boldsymbol{\eta}(t)$). Though exact knowledge of the disabled vessel's masa, Coriolis-centripetal, and damping matrices is not desirable, the focus of this work is more aligned with the study of the influence and compensation of other swarm vehicles on the disabled vessel dynamics.

Due to the swarm's decentralized architecture, a force allocation methodology [5] will not be feasible as each swarm vessel is not aware of the other's location, orientation, and thrust magnitude (i.e., the control force required for dynamic positioning and/or tracking will not be optimally distributed); therefore, the influence of other swarm vehicles will be viewed by each swarm member as a bounded disturbance and addressed through the development of a robust control structure.

VI. Control Design

After taking the time derivative of (8) and substituting in the translational kinematics of (5), the open-loop tracking error dynamics for $e_\psi(t)$ are given by the following expression

$$\begin{aligned} \dot{e}_\psi &= [0 \ 0 \ 1] \begin{bmatrix} -S(\psi)R^T\dot{\psi}(\boldsymbol{\eta} - \boldsymbol{\eta}_d) + \mathbf{v} \\ -R^T\dot{\boldsymbol{\eta}}_d \end{bmatrix} \\ &= [0 \ 0 \ 1] \begin{bmatrix} -S(\psi)\mathbf{e}_\eta + M^{-1}\mathbf{e}_v \\ + (M^{-1} - I_3)R^T\dot{\boldsymbol{\eta}}_d \end{bmatrix} \end{aligned} \quad (9)$$

where we have utilized the fact that $\dot{R}(\psi) = R(\psi)S(\dot{\psi})$, the skew-symmetric matrix $S(\dot{\psi}) \in \mathbb{R}^{3 \times 3}$ is defined in the following manner

$$S(\dot{\psi}) = \begin{bmatrix} 0 & \dot{\psi} & 0 \\ -\dot{\psi} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (10)$$

and the term $M^{-1}R^T\dot{\boldsymbol{\eta}}_d$ has been added and subtracted to the right hand side of (9), velocity tracking error signal $\mathbf{e}_v(t) \in \mathbb{R}^3$ has been defined in the following manner.

$$\mathbf{e}_v = M\mathbf{v} - R^T\dot{\boldsymbol{\eta}}_d. \quad (11)$$

After taking the time derivative of $\mathbf{e}_v(t)$ and substituting in the dynamics of (5), the open-loop linear velocity tracking error dynamics can be expressed

$$\dot{\mathbf{e}}_v = S(\dot{\psi})\mathbf{e}_v - C(\mathbf{v})\mathbf{v} - D\mathbf{v} + S(\dot{\psi})R^T\dot{\boldsymbol{\eta}}_d - R^T\ddot{\boldsymbol{\eta}}_d + \mathbf{F}_s + \mathbf{B}_i u_i. \quad (12)$$

where the definition of (11) has been utilized. Due to the decentralized approach, the term $\mathbf{B}\mathbf{u}$ of (5) has been separated into two components: i) the disturbance resulting from the influence from other swarm members $\mathbf{F}_s = \sum_{j=1, j \neq i}^{N-1} B_j u_j \in \mathbb{R}^3$ and ii) the control input of the i^{th} vehicle $\mathbf{B}_i u_i$. In addition, the swarm disturbance force is considered to be bounded in the sense that

$$\overline{F}_s \geq \|\mathbf{F}_s\|^2 \quad (13)$$

where \overline{F}_s represents a known upperbound on the force disturbance which can be approximated through summation of each swarm member's maximum thrust applied at a maximum radial length from the disabled vessel's center of mass. Future research will target the relaxation of knowledge of \overline{F}_s .

In order to simplify the control design, the filtered orientation tracking error signal $r_\psi(t) \in \mathbb{R}^1$ is defined in the following manner

$$\begin{aligned} r_\psi &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{r} \\ \mathbf{r} &= \mathbf{e}_v + \alpha \mathbf{e}_\eta \end{aligned} \quad (14)$$

where $\mathbf{r} \in \mathbb{R}^6$ denotes the 3-DOF filtered tracking error signal, $\alpha \in \mathbb{R}^1$ denotes a positive, scalar control constant. The open-loop filtered tracking error dynamics for $r_\psi(t)$ are formulated by taking the time derivative of (14), substituting in the open-loop expressions of (9) and (12) and can be expressed in the following form

$$\begin{aligned} \dot{r}_\psi &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \left[-S(\dot{\psi})\mathbf{r} - C(\mathbf{v})\mathbf{v} - D(\mathbf{v})\mathbf{v} \right. \\ &\quad \left. + S(\dot{\psi})R^T\dot{\boldsymbol{\eta}}_d - R^T\ddot{\boldsymbol{\eta}}_d + \mathbf{F}_s + \boldsymbol{\tau} + (\mathbf{B}_i u_i - \boldsymbol{\tau}) \right. \\ &\quad \left. - \alpha M^{-1}\mathbf{e}_v + \alpha(M^{-1} - I_3)R^T\dot{\boldsymbol{\eta}}_d \right] \end{aligned} \quad (15)$$

where $\boldsymbol{\tau}(t) \in \mathbb{R}^3$ has been added and subtracted to the right hand side of (15). Based on the structure of the open-loop system of (15) and the ensuing stability analysis, the 3-DOF control input vector $\boldsymbol{\tau}(t)$ is designed in the following manner

$$\begin{aligned} \boldsymbol{\tau} &= C(\mathbf{v})\mathbf{v} + D(\mathbf{v})\mathbf{v} - S(\dot{\psi})R^T\dot{\boldsymbol{\eta}}_d - R^T\ddot{\boldsymbol{\eta}}_d \\ &\quad + \alpha M^{-1}\mathbf{e}_v - \alpha(M^{-1} - I_3)R^T\dot{\boldsymbol{\eta}}_d - M^{-1}\mathbf{e}_\eta \\ &\quad - k_s r - k_r \dot{r} \end{aligned} \quad (16)$$

where $k_s, k_r \in \mathbb{R}^1$ denote positive, control constants. After substituting (16) into (15), the open-loop filtered tracking error dynamics can be rewritten as follows

$$\begin{aligned} \dot{r}_\psi &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \left[-S(\dot{\psi})r - \alpha M^{-1}\mathbf{e}_v \right. \\ &\quad \left. + \alpha(M^{-1} - I_3)R^T\dot{\boldsymbol{\eta}}_d \right. \\ &\quad \left. + (\mathbf{F}_s - k_n r) - k_r \dot{r} \right]. \end{aligned} \quad (17)$$

Since orientation tracking of the disabled vehicle is considered the primary objective, the swarm vehicle control input takes the form of the following expression

$$\begin{aligned} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} B_i u_i &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \boldsymbol{\tau} \\ (-l_{iy} \cos \alpha_i + l_{ix} \sin \alpha_i) u_i &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \boldsymbol{\tau}. \end{aligned} \quad (18)$$

In order to prevent loss of control influence into the rotational dynamics, the azimuth angle must be selected to avoid the following singularity condition

$$\alpha_i \neq \tan^{-1} \left(\frac{l_{iy}}{l_{ix}} \right). \quad (19)$$

Ensuring that the condition of (19) is satisfied, the swarm vehicle thrust input $u_i(t)$ can be calculated in the following manner

$$u_i = \frac{\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \boldsymbol{\tau}}{(-l_{iy} \cos \alpha_i + l_{ix} \sin \alpha_i)}. \quad (20)$$

VII. Stability Analysis

The swarm thrust input $u_i(t)$ of (20) guarantees that the rotation tracking error signal $e_\psi(t)$ is exponentially driven into an arbitrarily small neighborhood about zero in the sense that

$$\|e_\psi(t)\| \leq V(0) \exp\left(-\frac{\lambda_3}{\lambda_2} t\right) + \varepsilon \quad (21)$$

where $\lambda_3, \lambda_2, \varepsilon \in \mathbb{R}^1$ are positive, scalar constants (explicitly defined in the subsequent proof).

In order to illustrate the tracking result of (21), the following non-negative scalar function, denoted by $V(t)$, is defined as follows

$$V = \frac{1}{2} r_\psi^2 + \frac{1}{2} e_\psi^2 = \frac{1}{2} \mathbf{r}^T Q \mathbf{r} + \frac{1}{2} \mathbf{e}_\eta^T Q \mathbf{e}_\eta, \quad (22)$$

where the matrix $Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \in \mathbb{R}^{3 \times 3}$ and $V(t)$ can be upper and lower bounded by the following inequality

$$\lambda_1 \|\mathbf{z}\|^2 \leq V \leq \lambda_2 \|\mathbf{z}\|^2 \quad (23)$$

where $\mathbf{z} = \begin{bmatrix} \mathbf{r}^T & \mathbf{e}_\eta^T \end{bmatrix}^T \in \mathbb{R}^6$ and the constant parameters $\lambda_1, \lambda_2 \in \mathbb{R}^1$ are given by

$$\lambda_1 = \min \left\{ \frac{1}{2} \lambda_{\min}(Q) \right\}, \quad \lambda_2 = \max \left\{ \frac{1}{2}, \frac{1}{2} \lambda_{\max}(Q) \right\} \quad (24)$$

where $\lambda_{\min}\{Q\}$ denotes the minimum eigenvalue of the matrix Q . After taking the time derivative of (22), substituting in the closed-loop dynamics of (17) and (9), and cancelling common terms, the time derivative of $V(t)$ is given by the following expression

$$\begin{aligned} \dot{V} &= \mathbf{r}^T \left[-k_r \mathbf{r} + (\mathbf{F}_s - k_n \mathbf{r}) \right. \\ &\quad \left. + \mathbf{e}_\eta^T \left[-M^{-1} \alpha \mathbf{e}_\eta + \alpha(M^{-1} - I_3)R^T\dot{\boldsymbol{\eta}}_d \right] \right]. \end{aligned} \quad (25)$$

After applying the nonlinear damping argument [4] to the parenthetical term of (25), the time derivative of $V(t)$ can be further upperbounded in the following manner

$$\dot{V} = - \left(k_r - \frac{\overline{F_s}}{k_n} \right) \|\mathbf{r}\|^2 - \left(\alpha \lambda_{\min} \{QM^{-1}\} - \frac{\varepsilon_1^2}{2} \right) \|e_\eta\|^2 \quad (26)$$

where $\lambda_{\min} \{\cdot\}$ denotes the minimum and $\varepsilon_1, \varepsilon \in \mathbb{R}^1$ represent positive bounding constant defined as

$$\varepsilon \geq \frac{\|Q\alpha(M^{-1} - I_3)R^T\dot{\boldsymbol{\eta}}_d\|}{2\varepsilon_1}. \quad (27)$$

If the control gains are selected to ensure that

$$k_r \geq \frac{\overline{F_s}}{k_n}, \quad \alpha \geq \frac{\varepsilon_1^2}{2\lambda_{\min} \{QM^{-1}\}} \quad (28)$$

then the time derivative of $V(t)$ can be upperbounded in the following manner

$$\dot{V} \leq -\lambda_3 \|\mathbf{z}\|^2 + \varepsilon \leq -\frac{\lambda_3}{\lambda_2} V + \varepsilon \quad (29)$$

where $\lambda_3 \in \mathbb{R}^1$ is defined in the following manner

$$\lambda_3 = \min \left\{ \left(k_r - \frac{\overline{F_s}}{k_n} \right), \left(\alpha \lambda_{\min} \{QM^{-1}\} - \frac{\varepsilon_1^2}{2} \right) \right\}. \quad (30)$$

Linear arguments can be applied to (29) to obtain the exponential result of (21) in the sense that

$$\|e_\psi(t)\| \leq V(t) \leq V(0) \exp\left(-\frac{\lambda_3}{\lambda_2} t\right) + \varepsilon. \quad (31)$$

VIII. Conclusion

In this paper, a robust control strategy that achieves orientation tracking control of a disabled vessel through the utilization of a swarm of vehicles operating in a decentralized fashion has been presented. The control algorithm implemented on each swarm vehicle requires knowledge of the disabled vehicle's position and orientation. The influence of the other swarm vehicle was treated as a force disturbance into the model dynamics. Future research will consider the exploration of the integration of subtask objectives through techniques inspired from redundant robotic manipulator research [1] which may allow for such benefits as increased damping effects within the translational dynamics [2]. In the real system, it is difficult to estimate the attachment point of the swarm members and their relative orientation, making the \mathbf{B} matrix difficult to exactly compute. Therefore, another area of research interest is the development of an adaptive yaw tracking controller that is able to compensate for the unknown parameters associated with the input related matrix \mathbf{B} .

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