

Decentralized Cooperative Manipulation with a Swarm of Mobile Robots: the approach problem

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Abstract—In this paper we consider cooperative manipulation problems where a large group (swarm) of non-articulated mobile robots is trying to cooperatively control the velocity of some larger rigid body by exerting forces around its perimeter. We divide the problem into two phases: (1) approach; and (2) manipulation. This paper presents a novel decentralized solution to the approach phase. An artificial potential field is used to control the motion of the robots such that they establish contact with the object while avoiding collisions with other robots. The resulting grasp locally optimizes the group’s ability to resist disturbances. The approach is provably convergent and does not require centralized coordination of the robots. The results are illustrated in simulation.

I. INTRODUCTION

Robot swarms¹ are large groups of small, relatively unsophisticated, robots working in concert to achieve objectives that are beyond the capability of a single robot. One example of an application that can benefit from this approach is *non-prehensile cooperative manipulation*, where a group of non-articulated mobile robots transports a larger object in the plane, by applying forces to its perimeter. The advantages of the swarm are: (1) its ability to distribute applied forces over a large area, achieving an enveloping grasp on large objects; and (2) the maximum wrench the swarm can exert increases linearly as the number of swarm members increases. Applications include towing and material transport by ground-based mobile robots (Fig. 2); as well as marine applications (Fig. 1) involving autonomous tugboats such as towing disabled ships (ex. *U.S.S. Cole*), transporting components of large offshore structures (ex. oil platforms), or positioning littoral protection equipment (ex. hydrophone arrays).

We divide the problem into two phases: *approach* and *manipulation*. During the approach phase, robots must establish physical contact with the object while avoiding collision with other robots (see Fig. 3). This paper strictly addresses the approach phase. During the manipulation phase, robots exert forces on the object in an attempt to control its linear and angular velocity. The manipulation problem is addressed in our other work [4].

A. Related Work

The literature on traditional grasping and manipulation with multi-fingered robot hands is vast (see [17] for an

overview). Our work draws heavily from the grasp syntheses literature. Force closure grasps are not unique; so, once the closure criterion is met, a secondary *grasp quality function* can be defined. Common choices include “Max Normal Force” and “Min Analytic Center” [13], [14], which are important robustness measures for friction assisted grasping. In the case of the swarm manipulation problem considered here, we adopt the “Max Transfer” quality function, [11], [6], [16], [26], which measures the ability of the grasp to resist arbitrary net wrenches. Our approach of synthesizing a grasp by essentially applying some numerical optimization method to the quality function, has been used before (see [3], [9] and [14]).

However it is difficult to apply directly to large, decentralized swarms for two reasons. First, hand applications involve a small number of contacts (usually 2-3, but no more than 5 fingers), which frequently permit analytical solutions. Second, each of the contacts is controlled by a centralized decision maker, and centralized power supply.

Multi-robot cooperative object manipulation, both prehensile and non-prehensile ([21], [15]) has certainly been considered before in the context of small groups of robots (usually 1-3). Though it is generally unclear how to extend these to the distributed setting. Solutions involving larger groups are not generally: decentralized, *and* provably correct, *and* optimal with respect to a grasp quality function. For example the behavior-based frameworks in [10], [12], and [18] are decentralized but no attempt at formal grasp synthesis or proving controller convergence was made. While [22] and [1] do consider optimality criteria they do not offer decentralized solutions. The caging methods detailed in [23], [7], [2] and [19] are decentralized and provably correct; however, they provide no mechanism to include grasp quality functions, since they only consider form closure.

B. Contributions and Overview

We present a novel potential field control law, which simultaneously establishes contact with the object, avoids collision with other robots, and results in a grasping configuration that locally optimizes the group’s ability to resist disturbances. The approach is provably convergent and does not require centralized coordination of the robots. Our approach is inspired by the original work of [20], which creates a minima free potential field (a.k.a. Navigation function)

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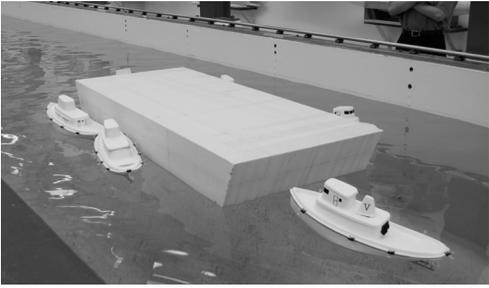


Fig. 1. Cooperative Manipulation. A group of 6 unmanned tugboats (0.5 meters long) and a scale model flat-bottomed barge (2 meters long). The tugs have articulated magnetic attachment devices used to grab the barge.



Fig. 2. Cooperative Manipulation. A group of iRobot Creates try to approach and push a box using a friction assisted grasp.

allowing a single robot to navigate to a goal location. We wish to extend this to a multi-robot setting that includes measures of grasp quality – similar to [24] and [8] which extend navigation functions to solve multi-robot formation control and *ad hoc* network quality problems, respectively.

We describe the robot and communication models in Sect. II. We present our solution to the approach phase in Sect. III. We adopt a decentralized version of the obstacle avoidance functions suggested in [24]. By extending ideas from [8] and [13] we are able to develop a novel objective function which reflects the notion of grasp quality *and* is suitable for multi-robot applications. The basic outline of our correctness proofs follows [20], [8], [24]. The emphasis of our proof is on showing that our new objective function possesses the required attributes to ensure almost everywhere convergence. In Sect. IV we provide simulation results and summarize contributions in V.

II. MODEL

Consider a planar disk-shaped workspace $\mathcal{W} = \{r \in \mathbb{R}^2 \mid \|r\| \leq R\}$ where R is a positive constant. The object to be manipulated is a rigid body, described by an open, convex, polygonal set $\mathcal{O} \subset \mathcal{W}$ (see Fig. 3). A body fixed reference frame is attached to the center of mass. N point robots populate the free space (though the method is easily extended to include disc-shaped robots). The position of robot i is $r_i \in \mathcal{W} - \mathcal{O}$, written in the object fixed reference frame.

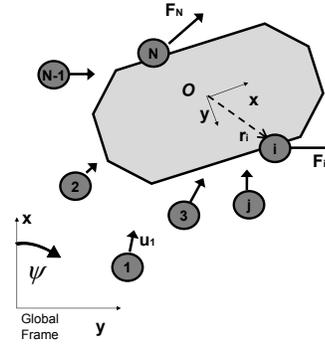


Fig. 3. Swarm Manipulation Approach Problem. N agents (dark circles) try to approach the object (shaded polygon) without colliding. The position of the i^{th} robot is defined by the vector r_i . During the approach phase robots are velocity controlled $u_i = v_i$. Once in contact they may apply a force F_i .

Assumptions: Initially none of the N robots are in contact with each other. Each robot knows the complete geometry of the object. And, at all times, each robot knows its own position, r_i .

A. Approach Phase Model

If a robot is outside the closure of \mathcal{O} , its motion is modeled according to the common kinematic equation $\dot{r}_i = u_i$. While this model does not address second order effects or nonholonomic constraints, it can be used to generate reference trajectories for lower level tracking controllers (see for example [5]) – especially at low speeds or for wheeled robots with zero turning radius.

B. Manipulation Phase Model

While we do not consider the manipulation phase in this paper, the contact model affects the notion of an optimal grasping configuration and therefore plays a role in the approach problem. If a robot is on the boundary of \mathcal{O} , it is considered to be in contact with the object. Once in contact, we assume the agents are *rigidly attached* to the object (their position is time invariant in the object-frame), and can apply an input force $F_i \in \mathbb{R}^2$ of any magnitude, in any direction (i.e. the applied force does not saturate and is not friction assisted). The model is relatively realistic for tugboats for example, who actually “tie up” to the barge rather than rely on friction (the apparatus in Fig. 1 uses magnetic attachment devices); or the robots in [25]. It is of course less representative for systems which use friction assisted grasps such as that shown in Figure 2. However, the framework in this paper applies in the case of more sophisticated contact models (see Sect. III and our companion work [3]).

C. Communication and Sensing Model

The robots are equipped with range limited sensing or communication devices that enable them to detect one another’s positions. Let $d(*,*)$ represent the distance function using the Euclidean metric, and $d_s > 0$ be the effective range of the sensor or wireless communication link. If $d(r_i, r_j) \leq d_s$

then robots i and j have knowledge of each other's positions. At any particular instant the set of robots within d_s of robot i is called its neighbors $\mathcal{N}_i = \{j \mid d(r_i, r_j) \leq d_s\}$

III. GRASPING CONTROLLER

The distributed multi-robot grasping problem is as follows.

Problem 3.1: Under the above modeling assumptions, design a kinematic motion control law, u_i , such that robot $i = [1, \dots, N]$

- 1) is guaranteed to not collide with any other robots or leave the workspace – i.e. $r_i \neq r_j$ and $\|r_i\| \leq R$, $\forall i \neq j \in [1, \dots, N], \forall t \in [0, \infty)$;
- 2) is guaranteed to asymptotically establish contact with the object (i.e. $r_i \rightarrow \partial\mathcal{O}$, as $t \rightarrow \infty$); and
- 3) maximizes the quality of the grasp subject to the previous two criteria (discussed later).

The ordering reflects the priority of the objectives.

Remark 3.2: Note that generating globally optimal grasping configurations, generally requires centralized knowledge of all robot's positions. We anticipate that due to the decentralized nature of the problem, the grasp quality function may only be locally optimized.

To this end we employ a Navigation Function-like framework [20], where each robot sets its velocity according to the gradient of some scalar potential function, $u_i = -k\nabla_i\phi_i(r)$, where $\nabla_i = \partial/\partial r_i$. However, our problem poses several challenges not addressed by the traditional navigation function framework, necessitating changes to the functions.

- The goal is not an isolated point, but rather a set of points that represents the boundary of the object.
- Any point on the boundary of the object that would cause interagent collision (not known in advance) must be excluded from the goal set.
- The concept of grasp quality must be reflected in the navigation function.
- Other robots act as (moving) obstacles, however in the traditional framework this would require centralized knowledge of every robot's position.

Traditionally Navigation functions are comprised of three functions: the obstacle function β ; the goal function γ ; and the navigation function itself $\phi(\gamma, \beta)$. In the following subsections we present our modifications to these functions.

A. Obstacle Functions

The function $\beta_i : \mathbb{R}^{2N} \rightarrow \mathbb{R}$ encodes the proximity of robot i to other robots or the workspace boundary. It is constructed from the individual obstacle functions [20]

$$\beta_i(r) = \prod_{j=0}^N \beta_{ij}. \quad (1)$$

The individual obstacle functions are designed such that $\beta_{ij}(r_i, r_j) \leq 0$ implies r_i is inside the obstacle, while $\beta_{ij}(r_i, r_j) > 0$ is in the free space. The boundary of the

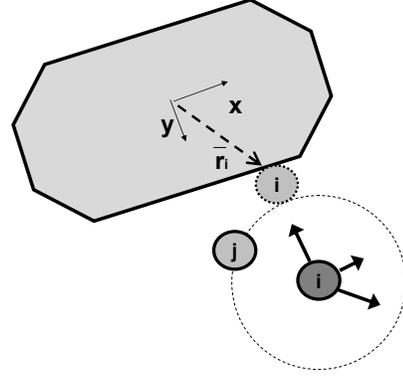


Fig. 4. Visualization of the gradients. $-\nabla_i d^2$ points along the inward normal of $\partial\mathcal{O}$. The grasp quality function g is computed by finding the projection along $\nabla_i d^2$, \bar{r}_i , and computing its distance from the origin. $\nabla_i g$ is tangential to $\partial\mathcal{O}$. Robot j , within d_s of i , causes $\nabla_i \beta_i$ to become repulsive.

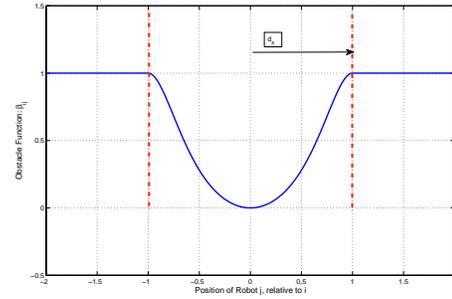


Fig. 5. Obstacle functions are saturated outside some range d_s .

workspace is obstacle 0

$$\beta_{0i} = \left(1 - \frac{(1+d_s^4)}{d_s^4} \frac{((R-\|r_i\|)^2 - d_s^2)^2}{((R-\|r_i\|)^2 - d_s^2)^2 + 1}\right) \frac{\text{sign}(d_s - R + \|r_i\|) + 1}{2} \quad (2)$$

and each robot acts as an obstacle to the other robots

$$\beta_{ij} = \left(1 - \frac{(1+d_s^4)}{d_s^4} \frac{(\|r_i - r_j\|^2 - d_s^2)^2}{(\|r_i - r_j\|^2 - d_s^2)^2 + 1}\right) \frac{\text{sign}(d_s - \|r_i - r_j\|) + 1}{2} \quad (3)$$

with d_s a (positive) sensing distance. The form of exponent of these functions was suggested in [24] as being suited to decentralized problems because (as seen in Fig. 5) the functions reach a constant, maximum value of 1 outside the range d_s —meaning that robots outside the sensing range will have zero contribution to the gradient.

B. Goal Function

The novelty of our approach is in the modification of the goal function $\gamma : \mathbb{R}^{2N} \rightarrow \mathbb{R}^+$ which takes the form

$$\gamma(r) = \sum_{i=1}^N d^2(r_i, \mathcal{O}) + \sum_{i=1}^N \sum_{j=i+1}^N c_{ij} + g \quad (4)$$

where the distance from a robot to the object is defined as $d(r_i, \mathcal{O}) = \min_{r \in \mathcal{O}} (\|r_i - r\|)$. The first term in this function takes its minimum of 0 when all robots are in contact with the object.

The terms c_{ij} are active when the interagent distance falls below some threshold d_{\min} , with the purpose of excluding from the goal set all points that result in interagent collisions [8]. ν is a positive constant.

$$c_{ij}(r_i, r_j) = \begin{cases} 0, & \|r_i - r_j\| > d_{\min} \\ \nu(d_{\min} - \|r_i - r_j\|)^3, & \|r_i - r_j\| \leq d_{\min} \end{cases} \quad (5)$$

Finally, the function g is a grasp quality function, many forms of which are suggested in the literature. We modify the *Max-Transfer* grasp quality function [6], [16] [14] defined as

$$q(r) = \min_{w \in \mathbb{R}^3} \max_{F_i} \frac{\|w\|}{\sum \|F_i\|}, \quad (6)$$

such that

$$w = \left[\sum F_i; \sum F_i \times r_i \right]$$

where the sum is over all robots in contact with the object ($r_i \in \partial\mathcal{O}$). In words, a large value of the quality function implies the swarm's is able to apply, or resist, large wrenches, w (in the worst-case direction) using small actuator forces F_i . Notice that while we do not explicitly consider actuator saturation, the quality function favors grasps which result in small applied forces.

Of course, selecting the appropriate norm on w is classically difficult due to a unit mismatch between forces and torques. However, some simplification of eq.(6) is possible. Since all norms are homogeneous, the ratio in eq.(6) is not affected by pure scaling of w or F_i . Therefore, with no loss in generality we can consider unit applied forces $\|F_i\| = 1$. While the maximization in eq.(6) requires force balance, the forces can be applied in any direction, therefore the discriminating factor is the length of the moment arms $\|r_i\|$. The idea of separately considering the force and torque components is referred to as the *Lexicographic* norm [6]. Therefore an equivalent definition of the Max-Transfer function is

$$q = \sum_{r_i \in \partial\mathcal{O}} \|r_i\|, \quad (7)$$

In order to fit into the navigation function framework, it must be differentiable – varying smoothly as robots establish contact with the object. For each robot r_i , we consider its projection onto the boundary of the obstacle, as in Fig. 4 (similar to [1] and [14])

$$\bar{r}_i = \arg \min_{r \in \partial\mathcal{O}} d(r_i, r) \quad (8)$$

The function also must take on a minimum of zero at globally optimal grasping configurations; but may possess other minima at locally optimal configurations.

Definition 3.3: Modified Max-Transfer Grasp Quality Function: Let

$$g = \sum_{i=1}^N (r_{\max} - \|\bar{r}_i\|)^2 \quad (9)$$

where r_{\max} is the maximum radius of the object.

Remark 3.4: It turns out that the critical features we exploit later in proving the correctness of the method are (1) orthogonality, $\nabla_i g \cdot \nabla_i d^2 = 0$, (2) differentiability, (3) nonnegativity, (4) boundedness over all $r \in \mathcal{W}$; (5) attaining a minima of $g = 0$ that coincides with the globally optimal grasping location; (6) (quasi-) convexity; and (7) separability. More complex grasp quality functions, such as those accounting for friction, will work equally well, provided they possess these properties.

C. Potential Function and Decentralized Control Law

The modified Navigation function [24] $\phi_i : \mathbb{R}^{2N} \rightarrow \mathbb{R}^+$ takes the form

$$\phi = \frac{\gamma(r)}{(e^\beta)^{1/\kappa}} \quad (10)$$

where $\kappa > 1$ is a tuning parameter which can be increased to “sharpen” the navigation function – removing any locally flat areas. The final controller law takes the well known form of [20]:

$$\dot{r}_i = -k \nabla_i \phi(r_1, \dots, r_N) \quad (11)$$

where k is a positive gain.

Regarding decentralization, note that all the terms in γ are additive, meaning that evaluating $\nabla_i \gamma$ only requires knowledge of this position of robots in \mathcal{N}_i . Also, because $\beta_{ij} = 1$ for any j not in \mathcal{N}_i , then $\nabla_i \beta_i$ can also be computed using only local information.

D. Proof of Correctness

We show that eq.(10) along with eq.(11) fulfills the criteria of Problem 4.1, because it only has minima at desirable goal locations and their basin of attraction is almost everywhere. The form of the proof follows the standard outline of [20].

- 1) All the minima of the goal function γ are on the free space portion of the boundary of the object to be manipulated; and, with proper selection of the parameter ν there are no other minima.
- 2) Minima of the goal function γ are also minima of ϕ_i .
- 3) There are no critical points of ϕ_i on the boundary of the free space.
- 4) With a sufficiently large κ all critical points can be pushed arbitrarily close to the obstacles.
- 5) These critical points are not minima nor degenerate.

Remark 3.5: Our potential function is not technically a navigation function because (1) its critical points are not isolated (i.e. it is degenerate) since many points on the perimeter of the object may be minima; (2) it is not uniformly maximal (i.e. $\phi = 1$) on the boundary. However we are most concerned with the almost everywhere convergence properties of the function.

Proposition 3.6: By selecting ν sufficiently large, all the minima of the goal function γ will be on the free space portion of the boundary of the object to be manipulated at local minima of g .

Proof:

$$\nabla_i \gamma = \nabla_i d^2(r_i, \mathcal{O}) + \nabla_i \sum_{j=i+1}^N c_{ij} + \nabla_i g \quad (12)$$

Consider two cases. First if no other robots are within d_{\min} of robot i , then $c_{ij} = 0$. The first term, $\nabla_i d^2(r_i, \mathcal{O})$, is normal to the surface of the object $\partial\mathcal{O}$, and only vanishes on its perimeter. The last term is, in general, tangential to $\partial\mathcal{O}$. Its minima occur when the projection of the robot is a vertex of the object. By orthogonality, the minima of γ can only occur where both these terms attain minima – on the vertices of the object.

In the second case, a robot is within $d < d_{\min}$ of another robot so $c_{ij} \neq 0$. Assume there was a critical point, then

$$(\nabla_i d^2 + \nabla_i g) = -\nabla_i c_{ij} \quad (13)$$

which implies that

$$\|\nabla_i d^2 + \nabla_i g\| = \|\nabla_i c_{ij}\|. \quad (14)$$

Then, the following bounds apply.

$$\begin{aligned} \sup \|\nabla_i d^2\| &= 2R \\ \sup \|\nabla_i g\| &= 2(r_{\max} - \|r_i\|) < 2R \\ \|\nabla_i c_{ij}\| &= \left\| 3\nu(d_{\min} - \|r_i - r_j\|)^2 \frac{(r_i - r_j)}{\|r_i - r_j\|} \right\| \\ &= 3\nu(d_{\min} - d)^2. \end{aligned}$$

Note then that by choosing

$$\nu > \frac{4R}{3(d_{\min} - d)^2} \quad (15)$$

we preclude the possibility of eq.(14), unless, of course, $\nabla_i c_{ij} = 0$, $\nabla_i g_i = 0$, and $\nabla_i d_i = 0$. ■

Proposition 3.7: Global minima of γ are minima of ϕ_i .

Proof:

$$\nabla_i \phi_i = \frac{1}{e^{\beta_i/\kappa}} \left[\nabla_i \gamma_i - \frac{\gamma_i}{\kappa} \beta_i^{1/\kappa-1} \nabla_i \beta_i \right] \quad (16)$$

At a global minima of γ , $\nabla_i \gamma = 0$ and $\gamma = 0$. Therefore the first and second additive terms in eq.(16) vanish. ■

Proposition 3.8: There are no critical points of ϕ_i on the boundary of the free space.

Proof: On the boundary of the freespace $\beta_i \rightarrow 0$. Therefore, since $\kappa > 1$ the second term in eq.(16) dominates

$$\nabla_i \phi_i = -\frac{1}{e^{\beta_i/\kappa}} \frac{\gamma}{\kappa} \beta_i^{1/\kappa-1} \nabla_i \beta_i. \quad (17)$$

By definition $\nabla_i \beta_i$ is the outward normal which points away from the obstacle surface in the direction of increasing β_i (the interior of the free space), since $\gamma \geq 0$, the coefficient is strictly negative. Therefore $-\nabla_i \phi_i$ points away from the obstacles, ensuring collision avoidance. Note that in the case when the obstacle is another robot $-\nabla_j \phi_j$ points in the opposite direction ensuring the separation velocity is sufficient. ■

Proposition 3.9: All critical points can be pushed arbitrarily close to the boundary with sufficiently large κ .

Proof: At a critical point $\nabla_i \phi_i = 0$, so eq.(16) becomes

$$\nabla_i \gamma = \frac{\gamma}{\kappa} (\beta_i)^{1/\kappa-1} (\nabla_i \beta_i). \quad (18)$$

For points not in the goal set

$$\kappa \beta^{(\kappa-1)/\kappa} = \gamma_i \frac{\nabla_i \beta_i}{\nabla_i \gamma} \quad (19)$$

Since $\nabla_i \gamma$ and $\nabla_i \beta_i$ are bounded within the freespace. If κ is made arbitrarily large, the value of β_i at which a critical point occurs must decrease. Therefore the critical points can be pushed arbitrarily close to the boundary ($\beta_i = 0$). ■

Proposition 3.10: All other critical points are neither minima nor degenerate.

Sketch of Proof: The proof is omitted for brevity. In [24] they prove that using eq.(10) and (1) this follows, provided γ is quasi-convex. Here, γ is constructed from a sum of quasi-convex functions in eqs.(10), (9), and (5); ensuring its quasi-convexity.

IV. SIMULATIONS

Consider Figure 6. A group of 20 robots converges on a rectangular object. We set $d_s = 1$, $d_{\min} = 0.25$, $\nu = 10$, $k = 0.1$ and $\kappa = 2$. The corners represent optimal grasping locations (local minima of g); however, robots must maintain a 0.25 unit spacing. In Figure 7 one can see that the potential function is continually decreasing; while, according to the standard definition, the max-transfer grasp quality function is continually increasing (improving).

Figure 8 depicts a similar scenario. This time 10 robots begin in a crowded circular region, the minimum allowed spacing is reduced to $d_{\min} = 0.1$ and the obstacle's long axis is exaggerated. As the two robots beginning closest to the object begin to head for the far vertices, the remaining robots are pushed outward in order to avoid collisions, eventually getting pushed to the far facets.

V. CONCLUSION

In this paper we consider cooperative manipulation problems where a large group (swarm) of non-articulated mobile robots is converging around some larger rigid body with the goal of manipulating it. We present a novel potential field control law, which simultaneously establishes contact with the object, avoids collision with other robots, and results in a grasping configuration that locally optimizes the groups ability to resist disturbances. While other work has appeared in this area, we believe this is the first provably-correct, decentralized control law that includes a measure of grasp quality. The approach follows a navigation-function-like framework which extends some recent developments on decentralized navigation functions. The primary technical novelty involves introducing a modified objective function and proving that the potential field retains its almost everywhere convergence properties. While our grasp quality function is simplified, we extract the 7 essential properties required by the framework. Extending this to other quality functions, based on friction-assisted grasping models is a topic of ongoing work.

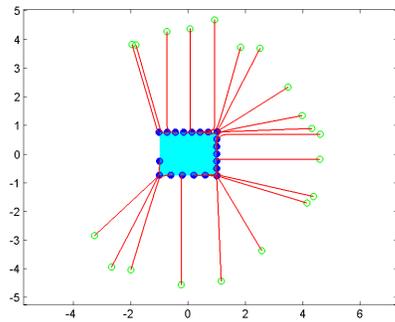


Fig. 6. Simulation of the Approach Controller. A group of 20 robots converge on a rectangular object. The corners represent optimal grasping locations; however, robot must maintain a 0.25 unit spacing.

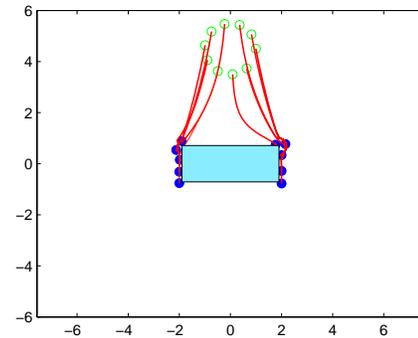


Fig. 8. Simulation of the Approach Controller. A group of 10 robots begin crowded together. The optimal grasping locations are on the facets perpendicular to the long axis of the object.

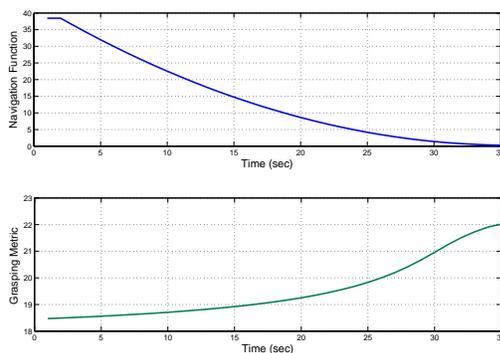


Fig. 7. Simulation of the Approach Controller. (Top) The potential function is continually decreasing. (Bottom) the traditional version of the max-transfer grasp quality function is continually increasing (improving).

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