Teleoperation of Multi-Agent Systems with Nonuniform Control Input Delays

Erick J. Rodríguez-Seda†
School of Engineering and Computer Science,
University of Texas, Dallas, TX 75080, USA

Dušan M. Stipanović
Coordinated Science Laboratory,
University of Illinois, Urbana, IL 61801, USA

Mark W. Spong
School of Engineering and Computer Science,
University of Texas, Dallas, TX 75080, USA

Thursday 9th February, 2012

Abstract

Consensus and coordination of multiple teleoperated agents has been a control topic of growing interest over the last decade due, in part, to its many potential applications and related complex challenges. One of these challenges is to guarantee stability and motion coordination in the presence of inherent communication and input delays as well as system nonlinearities in the multi-agent system. Addressing this challenge herein, we report on a model reference robust control framework that guarantees stability, motion coordination, and formation control of $N$ output strictly passive, nonlinear Lagrangian systems with arbitrarily large, nonuniform control input delays. The control framework is comprised of a reference model coupled to the nonlinear systems via the use of $N$ modified scattering transformation blocks. It is shown that the overall control architecture is stable for any large constant delay, robust to system uncertainties, and that the control parameters are delay-independent. A numerical example with four nonlinear planar manipulators and another example with six ground vehicles are finally presented to illustrate the performance of the proposed controller.

*This work was partially supported by the National Science Foundation Grant ECCS 07-25453 and by the University of Texas at Dallas.
†Corresponding author. E-mail: erodriguez@utdallas.edu
1 Introduction

Recent advances in electronics, computation, and integrated communications have allowed multiple distributed autonomous systems to solve diverse control problems in a cooperative manner \cite{27, 38}. Automated tasks that were once carried by a single, complex machine can now be performed faster and more efficiently by a network of physically disconnected, cooperative systems (which we will also refer to as agents). In fact, agents in a networked multi-robotic system can asynchronously operate from remote locations and share information through a wireless communication network while distributing the workload (e.g., space, objectives, and computation) of a common task. This distributed configuration mitigates implementation and maintenance costs and provides scalability, redundancy, and robustness in the control process. Such advantages make remotely operated multi-agent systems appealing for several applications, including space and oceanic exploration \cite{18, 2}, search and rescue \cite{15}, coverage control \cite{7}, intelligent transportation systems \cite{22}, structural health monitoring \cite{3}, power distribution networks \cite{25, 32}, and teleoperation \cite{37}.

One of the foremost challenges in the teleoperation (i.e., remote control) of multi-agent systems is to achieve coordination and consensus independently of communication and input delays. The use of communication networks to remotely operate multiple agents inevitably introduces delays into the control process due, for the most part, to large distances among agents and control components, slow sampling rates, and congested networks. Such delays are well known to cause performance degradation and, in the worst scenario, instability of the overall system. Therefore, several researchers have investigated and designed multiple control solutions over the last decade to overcome these delay-induced problems. For instance, the consensus problem for a group of linear systems with nonzero communication delay (i.e., agent-to-agent) has been solved, in separate, by \cite{17, 26, 20} for the case of zero input delay (i.e., controller-to-plant), and by \cite{14, 13} for the case of delayed self-position information (yet, the latter efforts assume that velocity information is non-delayed and, consequently, the controller can inject artificial damping into the system with the aim of stabilizing it). Likewise, the coordination of multiple agents in leaderless and leader-following configurations \cite{10} with communication and input delays has been addressed in \cite{30, 19, 31, 14, 47}, for systems with single-integrator dynamics, and in \cite{40, 39, 21, 24}, for groups of double-integrators.

The research efforts cited above are examples of control solutions formulated for linear systems with communication and/or input delays. For the case of nonlinear systems, control algorithms are rare in the literature \cite{33}. Some of the few research examples that solve the consensus problem for cooperative nonlinear systems with communication delays (i.e., delays between pairs of agents) include...
where passivity-based properties of a wide class of nonlinear systems are exploited, and where sufficient conditions for synchronization of nonlinear Lagrangian systems are derived via the use of contraction analysis. Similarly, the synchronization problem between two serially connected nonlinear systems with communication and/or input delays has been attained in. However, to the best of our knowledge, the question of how to coordinate multiple nonlinear agents with pure time-delay control inputs remains relatively unaddressed.

Motivated by the need for control solutions for nonlinear systems, we now introduce a passivity-based Model Reference Robust Control (MRRC) framework that guarantees coordination (i.e., tracking and formation control) of a group of nonlinear Lagrangian systems with nonzero input delays. The proposed distributed control architecture stems from the work reported in, where the use of a modified version of the wave scattering transformation, widely used in teleoperation and recently applied to networked control systems, is employed to achieve stability and full-state convergence of a single nonlinear system with input delays. Herein, we extend the passivity-based MRRC framework to the general case of multiple heterogeneous nonlinear Lagrangian systems with nonuniform input delays. The control formulation builds on the assumption that the nonlinear agents are output strictly passive in order to guarantee state convergence and formation control independently of any large constant input delays as well as dynamics uncertainties. Moreover, the control parameters are shown to be delay-independent. Two simulation examples, one comprising four planar manipulators and another comprising six unmanned vehicles, are presented illustrating the performance of the proposed control architecture under constant control input delays.

2 Preliminaries

We consider a group of \( N \) \( n \)-degree-of-freedom (DOF) nonlinear Lagrangian systems with equations of motion given by

\[
M_i(q_i(t))\ddot{q}_i(t) + C_i(q_i(t), \dot{q}_i(t))\dot{q}_i(t) + \frac{\partial F_i(\dot{q}_i(t))}{\partial q_i(t)} = u_i(t - T_i)
\]
where $q_i(t) \in \mathbb{R}^n$ are the vectors of generalized coordinates, $M_i(q_i(t)) \in \mathbb{R}^{n \times n}$ are the positive definite inertia matrices, $C_i(q_i(t), \dot{q}_i(t)) \in \mathbb{R}^{n \times n}$ are the matrices of Coriolis and centrifugal terms, $u_i(t - T_i) \in \mathbb{R}^n$ are the control inputs, and $F_i(q_i(t)) \in \mathbb{R}$ are the Rayleigh dissipation functions for $i \in \{1, \cdots, N\}$. The control inputs are considered to be state-feedback laws depending on the delayed agents’ velocities, $\dot{q}_i(t - T_i)$, where $T_i \geq 0$ represent the round-trip input delays. We assume that $T_i = T_{1i} + T_{2i}$, where $T_{1i} \geq 0$ and $T_{2i} \geq 0$ denote the constant $i$th-agent-to-controller and controller-to-$i$th-agent delays, respectively (see Figure 1). We also assume that the dissipative forces are upper-bounded whenever $\dot{q}_i \in L_\infty$ and that the following inequality is satisfied:

$$\dot{q}_i^T \frac{\partial F_i(q_i)}{\partial \dot{q}_i} \geq \rho_i \| \dot{q}_i \|^2,$$

for some $\rho_i > 0$, (2)

Similarly, we suppose that the inertia matrices as well as the $C$ matrices satisfy the following properties:

**Property 1** (Skew-Symmetry). The matrices $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ are skew-symmetric.

**Property 2** (Uniform Boundedness). $\exists$ positive constants $\lambda_i$ and $\overline{\lambda}_i$ such that $\lambda_i I \leq M_i(q_i) \leq \overline{\lambda}_i I$, where $I \in \mathbb{R}^{n \times n}$ is the identity matrix.

**Property 3** (Boundedness of C-matrix). $\exists$ positive constants $k_{C_i}$ such that $\| C_i(q_i, \dot{q}_i) \| \leq k_{C_i} \| \dot{q}_i \|$.

**Remark 1.** The skew-symmetric property is attained if the $jk$th entries of the $C_i$ matrices are defined using the Christoffel symbols of the first kind, i.e.,

$$C^{jk}_{i}(q_i, \dot{q}_i) = \sum_{l=1}^{n} \frac{1}{2} \left[ \frac{\partial M_{ij}^{lk}}{\partial q_l} + \frac{\partial M_{ij}^{kl}}{\partial q_l} - \frac{\partial M_{ij}^{lk}}{\partial q_l} \right] \dot{q}_l^l. \quad (3)$$

The second property, although more restrictive, is satisfied by many different configurations of robotics systems [11]. For instance, it trivially holds when the system has linear dynamics. It also holds for nonlinear robotic manipulators with 1) only revolute joints; 2) only prismatic joints; 3) a series of prismatic joints followed by a series of revolute joints; or 4) the axis of translation of each prismatic joint parallel to all preceding revolute joints. Similarly, many nonlinear Lagrangian vehicles can be shown to possess bounded inertia matrices if the states of the system are chosen properly [8, 9]. Finally, the third property is a direct result of Property 2.

We now introduce the concept of passivity, which will play a central role in the construction and analysis of the control framework.

---

2In what follows, we will omit time dependence of signals when necessary to avoid cluttered equations.
Definition 1. A system with input $u$ and output $y$ is said to be passive if
\[ \int_0^t y^T u d\theta \geq -\kappa + \nu \int_0^t u^T u d\theta + \rho \int_0^t y^T y d\theta \] (4)
for some nonnegative constants $\kappa, \nu,$ and $\rho$. Moreover, it is said to be lossless if equality persists and $\nu = \rho = 0$, input strictly passive if $\nu > 0$, and output strictly passive if $\rho > 0$.

Theorem 1. The $i$th Euler-Lagrange system in (1) is output strictly passive with respect to input $u_i$ and output $\dot{q}_i$.

Proof. Consider the following positive definite function
\[ H_i = \frac{1}{2} \dot{q}_i^T M_i(q_i) \dot{q}_i \geq 0. \]

By taking the time derivative of $H_i$ we obtain
\[ \dot{H}_i = \dot{q}_i^T M_i(q_i) \dot{q}_i + \frac{1}{2} \dot{q}_i^T M_i(q_i) \dot{q}_i \]
\[ = \dot{q}_i^T \left( u_i - C_i(q_i, \dot{q}_i) \dot{q}_i - \frac{\partial F_i(\dot{q}_i)}{\partial \dot{q}_i} \right) + \frac{1}{2} \dot{q}_i^T M_i(q_i) \dot{q}_i \]
and recalling Property 1 we have
\[ \dot{H}_i = \dot{q}_i^T u_i - \dot{q}_i^T \frac{\partial F_i(\dot{q}_i)}{\partial \dot{q}_i}. \] (5)

Now, integrating both sides of (5) yields
\[ \int_0^t \dot{q}_i^T u_i d\theta = \int_0^t \dot{q}_i^T \frac{\partial F_i(\dot{q}_i)}{\partial \dot{q}_i} d\theta + H_i(t) - H_i(0) \]
\[ \geq \rho_i \int_0^t \dot{q}_i^T \dot{q}_i d\theta - H_i(0) \] (6)
where we have used (2) and the fact that $H_i(\theta) \geq 0 \forall \theta \Rightarrow H_i(t) - H_i(0) \geq -H_i(0)$. Then, by choosing $\kappa = H_i(0)$ and $\rho = \rho_i$, we finally conclude that (1) is output strictly passive.

3 Control Framework

3.1 Control Objectives

As part of the control objectives, we identify two primary goals. First, the overall system must be stable, meaning that all trajectories must remain bounded; and second, the agents must converge to the desired state (e.g., position and orientation) within the formation, whereas the formation’s centroid must converge to the desired trajectory. Mathematically, we would like $q_i(t) \rightarrow q_d + \gamma_i$.
∀i ∈ {1, · · · , N} and \( \frac{1}{N} \sum_{i=1}^{N} q_i(t) \to q_d \) as \( t \to \infty \), where \( q_d \in \mathbb{R}^n \) is the desired trajectory of the formation’s centroid and \( \gamma_i \in \mathbb{R}^n \) is the desired relative distance and orientation of the \( i \)th agent with respect to the center of the formation and satisfying, without loss of generality, \( \sum_{i=1}^{N} \gamma_i = 0 \). In what follows, we will assume that \( q_d \) and \( \gamma_i \) are constant. Moreover, we assume that \( q_d \) and \( \gamma_i \) are unknown to the agents.

### 3.2 Passivity-Based MRRC Framework for Multi-Agent Systems

In order to achieve the control objectives, we propose the use of a distributed, passivity-based control framework. The control framework is comprised of a reference model coupled to the time delay nonlinear agents in (1) via the implementation of \( N \) scattering transformation blocks. A pictorial representation is given in Figure 2.

For simplicity, we take the reference model to be a linear system described by

\[
\begin{align*}
\ddot{q}_m(t) &= A_m \dot{q}_m(t) + u_m(t) + r_m(t) \\
y_m(t) &= \dot{q}_m(t)
\end{align*}
\]

where \( q_m(t), \dot{q}_m(t) \in \mathbb{R}^n \) are the state vectors, \( y_m(t) \in \mathbb{R}^n \) is the output vector, \( u_m(t) \in \mathbb{R}^n \) is the control input, and \( A_m \in \mathbb{R}^{n \times n} \) is a symmetric Hurwitz matrix. The reference signal \( r_m(t) \in \mathbb{R}^n \) is governed by

\[
r_m(t) = K_d(q_d - q_m(t))
\]

where \( K_d \in \mathbb{R}^{n \times n} \) is a positive-definite constant matrix.

The scattering transformation blocks are employed to stabilize the coupling between the reference model and the nonlinear agents. The scattering transfor-
The transmission equations are given as

\[ \tau_m(t) = b_i \dot{e}_m(t) + K_m e_m(t) \]  
\[ w_m(t) = \sqrt{\frac{2}{b_i}} \tau_m(t) - v_m(t) \]  
\[ \dot{q}_{md}(t) = \frac{1}{b_i} \left( \tau_m(t) - \sqrt{2b_i} v_m(t) \right) \]  
\[ q_{md}(t) = \int_0^t \dot{q}_{md}(\theta) d\theta \]  
\[ e_m(t) = q_m(t) - q_{md}(t) \]  
\[ \tau_i(t) = \sqrt{2b_i} w_i(t) \]  
\[ v_i(t) = w_i(t) - \sqrt{2b_i} \dot{q}_i(t) \]  

where \( b_i \), namely the wave impedances, are positive constants, \( K_m \) are symmetric positive definite matrices, \( q_{md} \in \mathbb{R}^n \) are internal states of the controller, and

\[ v_m(t) = v_i(t - T_{1i}) \]  
\[ w_i(t) = w_{m_i}(t - T_{2i}) \]

are known as the transmission equations. The control inputs to the reference model and the agents are then given as \( u_m(t) = -\tau_m(t) \) and \( u_i(t - T_i) = \tau_i(t) \) for \( i \in \{1, \cdots, N\} \), respectively, where

\[ \tau_m(t) = \sum_{i=1}^N \tau_{m_i}(t). \]  

For sake of clarity, Figure 3 illustrates the implementation of the scattering transformation equations.

The appeal of using the scattering transformation comes from its ability to make the communication channel passive independent of any arbitrary large constant round-trip delays. To demonstrate this statement, let us verify that the communication channel for each agent has, indeed, been made passive. Manipulating (9) to (17), we can easily show that

\[ \dot{q}_T \]  
\[ q_{md}(t) - q_i^T (\tau_i - b_i \dot{q}_i) = \frac{1}{2} \left( w_{m_i}^T w_{m_i} - w_i^T w_i + v_i^T v_i - v_{m_i}^T v_{m_i} \right) \]

3The same results hold if \( b_i \) are positive definite matrices.

4In practice, the wave variables \( v_i \) and \( w_{m_i} \) are not transmitted nor communicated between the reference model and the \( i \)th agent. Instead, \( \dot{q}_i \) and \( \tau_i \) are the variables being exchanged over the communication channels [see Figure 3]. The variables \( v_i \) and \( w_{m_i} \) are constructed using the delayed values of \( \dot{q}_i \) and \( \tau_i \) such that (10) and (14) hold. They are defined as in (10) and (14) for convention.

5Note that, after some manipulation of the scattering variables, we can show that \( \tau_i(t) = u_i(t - T_i) = 2\tau_{m_i}(t - T_{2i}) - \sqrt{2b_i} w_{i}(t - T_i) + 2b_i \dot{q}_i(t - T_i) \), which explicitly exhibits the delayed nature of the control law as well as the dependence on the delayed state \( \dot{q}_i(t - T_i) \).
Then, integrating the above equation with respect to time yields
\[
\int_0^t \left( \dot{q}_{md}^T \tau_{md} - \dot{q}_i^T (\tau_i - b_i \dot{q}_i) \right) d\theta = \frac{1}{2} \int_{t-T_2i}^t w_{mi}^T w_{mi} d\theta + \frac{1}{2} \int_{t-T_1i}^t v_i^T v_i d\theta \geq 0
\]
which confirms the passivity claim. The lower bound in (19) implies that the energy is temporarily stored in the transmission lines and, therefore, that the communication channel is passified independently of the size of $T_1$ and $T_2$.

**Remark 2.** The definition of the scattering transformation proposed here differs from its standard implementation (e.g., [5, 28, 23]) in the sense that the current states of the time delay nonlinear agents are assumed to be inaccessible to the agent’s local control loop and, therefore, cannot be used when computing the transformation variables. Consequently, all scattering transformation variables are computed at the same location in the network (at the reference model’s site), as opposed to their conventional bisected (or mirror) implementation. See Figure 3.

**Remark 3.** Although the agents do not directly share state information among each other, the state of the $i$th agent affects the behavior of the reference model, which, in turn, directly affects the state and command of all other agents. Thus, any change or perturbation in the $i$th agent’s state will eventually propagate to the other agents. In this sense, the coordination and formation control problem is solved cooperatively.

**Remark 4.** The proposed controller implicitly assumes that the delayed wave variables $w_{mi}(t - T_i)$ are known or that, at least, they can be decoded from the
outputs of the multi-agent system. Specifically, we assumed that \( w_m(t-T_i) \) are available when computing (15) and (16), i.e.,

\[
v_m(t) = v_i(t-T_i) = w_m(t-T_i) - \sqrt{2b_i} \dot{q}_i(t-T_i).
\]

In the case that \( w_m(t-T_i) \) are unknown or unavailable, knowledge of the round-trip delay values are required to reconstruct \( w_m(t-T_i) \). However, it is worth mentioning that knowledge of the delays or \( w_m(t-T_i) \) is not required in the design of the reference model and the scattering transformation parameters, that is, \( A_m, K_d, b_i, \) and \( K_m \) are all independent of \( T_1 \) and \( T_2 \).

4 Stability and Formation Control

We now proceed to claim the attainment of the control objectives by the proposed controller. The following theorem will be used to establish stability of the overall system independently of any initial condition and constant delays, whereas its corollary will guarantee state convergence and formation control.

**Theorem 2.** Consider the multi-Lagrangian system (1) coupled to the reference model via the scattering transformation blocks and let \( 0 < b_i < \rho_i \) for \( i = \{1, \ldots, N\} \). Then, for all \( i \) and all initial conditions, we have the following results.

i. The signals \( \dot{q}_m(t), \dot{q}_i(t), e_m(t), \dot{e}_m(t), \dot{q}_m(t), \dot{q}_i(t), \) and \( \ddot{q}_m(t), \ddot{q}_i(t) \) are bounded \( \forall t \geq 0 \) and the velocity terms \( \dot{q}_m(t), \dot{e}_m(t), \) and \( \dot{q}_i(t) \) converge to zero.

ii. The error signals \( e_m(t) \) and \( q_m(t) - q_d \) converge asymptotically to zero.

**Proof.** Consider the group of agents described by (1) and coupled via the scattering transformation equations (9) to (17). Let the following Lyapunov candidate function be given by

\[
\mathcal{V} = \sum_{i=1}^{N} \left( \mathcal{H}_i + \frac{1}{2} e_m^T K_m e_m \right) + \frac{1}{2} (q_d - q_m)^T K_d (q_d - q_m)
\]

\[
+ \frac{1}{2} \dot{q}_m^T q_m + \sum_{i=1}^{N} \int_0^t \left( \dot{q}_{md}^T \tau_m - \dot{q}_i^T (\tau_i - b_i \dot{q}_i) \right) d\theta.
\]

(20)
Then, taking the time derivative of (20) yields

$$
\dot{V} \leq \sum_{i=1}^{N} \left( -\rho_i \dot{q}_i^T \dot{q}_i + \dot{q}_i^T \tau_i + \dot{e}_m^T K_m e_m \right) - \dot{q}_m^T K_d (q_d - q_m)
+ q_m^T (A_m q_m - \tau_m + K_d (q_d - q_m)) + \sum_{i=1}^{N} \left( q_{md}^T \tau_{mi} - \dot{q}_i^T \tau_i + b_i \dot{q}_i^T \dot{q}_i \right) \leq -\sum_{i=1}^{N} (\rho_i - b_i) \dot{q}_i^T \dot{q}_i + \sum_{i=1}^{N} \dot{e}_m^T K_m e_m + q_m^T A_m q_m
- \sum_{i=1}^{N} \dot{q}_m^T (b_i e_m + K_m e_m) + \sum_{i=1}^{N} \dot{q}_{md}^T (b_i \dot{e}_m + K_m e_m) \leq -\sum_{i=1}^{N} (\rho_i - b_i) \dot{q}_i^T \dot{q}_i - \mu q_m^T q_m - \sum_{i=1}^{N} b_i \dot{e}_m^T \dot{e}_m
$$

where \( \mu > 0 \) is the smallest eigenvalue of \(-A_m\). Since \( b_i < \rho_i \forall i \), we have that

$$
\dot{V} \leq -\mu \| q_m \|^2 - \sum_{i=1}^{N} (\rho_i - b_i) \| \dot{q}_i \|^2 - \sum_{i=1}^{N} b_i \| e_m \|^2 \leq 0
$$

(21)

and, therefore, the overall multi-agent system is stable in the sense of Lyapunov. Moreover, we can invoke LaSalle’s Invariance Principle for delay systems [12] and conclude that \( q_m, \dot{q}_i, e_m, \) and \( q_{md}, \dot{e}_m \) converge to zero for all \( i \in \{1, \ldots, N\} \).

Now, in order to demonstrate boundedness of all signals, let us consider once again (21). Integrating at both sides of (21) we obtain that \( V(t) \leq V(0) < \infty \), which implies that \( q_m, q_{md}, e_m, q_i, \dot{q}_i \) are all bounded. Similarly, from the scattering transformation equations (9) to (13) and the transmission equation (17), we can easily verify that

$$
\tau_i(t) = b_i \dot{q}_m(t - T_2_i) + K_m e_m(t - T_2_i)
$$

(22)

is also bounded \( \forall i \). Then, substituting (22) and (10) into (11) yields

$$
2b_i q_{md}(t) = b_i \dot{q}_m(t) + K_m e_m(t) + 2b_i \dot{q}_i(t - T_{1_i})
- b_i \dot{q}_m(t - T_i) - K_m e_m(t - T_i)
$$

(23)

and, therefore, \( q_{md}, \dot{e}_m \) are bounded. Similarly, rewriting (11) gives us that

$$
\tau_{mi}(t) = \frac{b_i}{2} (\dot{q}_m(t) + \dot{q}_m(t - T_i))
+ \frac{K_m}{2} (e_m(t) + e_m(t - T_i)) - b_i \dot{q}_i(t - T_{1_i})
$$

(24)

Then, the boundedness of \( e_m(t), q_m, \) and \( \dot{q}_i \) implies that \( \tau_{mi} \in \mathcal{L}_\infty \), and from the definition of the reference model (7) and (18), we obtain that \( \dot{q}_m \) is also...
bounded. Furthermore, integrating (23) with respect to time yields
\[
\int_0^t \dot{q}_{md}(\theta) d\theta = \int_0^{t-T_i} \dot{q}_i(\theta) d\theta + \frac{1}{2} \int_{t-T_i}^t \left( q_m(\theta) + \frac{K_m}{b_i} e_m(\theta) \right) d\theta. \tag{25}
\]
Rearranging the above equation, we obtain that
\[
q_i(t-T_i) = q_i(0) + q_{md}(t) - q_{md}(0) + q_m(t)
- q_m(t-T_i) + \frac{K_m}{2b_i} \int_{t-T_i}^t e_m(\theta) d\theta.
\tag{26}
\]
Since all terms in (26) are bounded, we can conclude that \( q_i \in L_\infty \).

Now, let us solve (1) for \( \ddot{q}_i \) as
\[
\ddot{q}_i = M_i^{-1} q \left( \tau_i - \frac{\partial F_i(q)}{\partial \dot{q}_i} - C_i(q, \dot{q}_i) \dot{q}_i \right), \tag{27}
\]
where \( M_i^{-1} \) exist and are bounded \( \forall i \) due to Property 2. Similarly, from boundedness of \( \dot{q}_i \), we have that \( C_i \) (from Property 3) and \( \frac{\partial F_i(q)}{\partial \dot{q}_i} \) are bounded, which, in addition to boundedness of \( \tau_i \), lead us to the conclusion that \( \ddot{q}_i \in L_\infty \) \( \forall i \in \{1, \cdots, N\} \).

To prove the second statement in the theorem, let us consider (24) rearranged as
\[
\tau_m(t) = \frac{b_i}{2} (\dot{q}_m(t) + \dot{q}_m(t-T)) + \frac{K_m}{2} \int_{t-T_i}^t e_m(\theta) d\theta - b_i \dot{q}_i(t-T_i). \tag{28}
\]
Due to the fact that all signals on the right-hand side of (28) go to zero, we have that \( \tau_m \to 0 \), which similarly implies that \( e_m \to 0 \). Then, computing the time derivative of (28) yields that
\[
\dot{\tau}_m(t) = \frac{b_i}{2} (\ddot{q}_m(t) + \ddot{q}_m(t-T)) + \frac{K_m}{2} (\ddot{e}_m(t) + \ddot{e}_m(t-T_i)) - b_i \ddot{q}_i(t-T_i).
\tag{29}
\]
Noticing that all signals on the right-hand side of (29) are bounded, we obtain that \( \dot{\tau}_m(t) \) are also bounded. Likewise, by taking the time derivative at both sides of (7), we can easily verify that \( \ddot{q}_m \) is also bounded. Then, since \( \int_0^t \dot{q}_m(\theta) d\theta \to -q_m(0) < \infty \) as \( t \to \infty \), we can apply Barbalat’s Lemma and conclude that \( \ddot{q}_m \to 0 \). Finally, using (7) and the convergence results for \( q_m, \dot{q}_m, \) and \( \tau_m \), we obtain that \( r_m \to 0 \Rightarrow q_m - q_d \to 0 \), which completes the proof.

Theorem 2 establishes global asymptotic stability and boundedness of the states for a group of nonlinear dissipative Lagrangian systems with arbitrarily large constant input delays as well as convergence of the reference model to the desired formation’s centroid, i.e., \( q_m(t) \to q_d \). Yet, it does not guarantee
convergence of the agents’ positions to the desired states, i.e., \( q_i \to q_d + \gamma_i \), and convergence of the formation’s centroid to the desired geometric center, i.e., \( \frac{1}{N} \sum_{i=1}^{N} q_i(t) \to q_d \) (it can be shown, however, that they converge to a constant value). In the following we provide sufficient conditions for the latter objectives.

**Corollary 1.** If \( q_{md,i}(0) = q_i(0) - \gamma_i \forall i \in \{1, \cdots, N\} \), then \( q_i(t) \to q_d + \gamma_i \) and \( \frac{1}{N} \sum_{i=1}^{N} q_i(t) \to q_d \) as \( t \to \infty \).

To prove the above statement, let us consider (25). From Theorem 2 we have that \( \dot{q}_{md}(t) \) and \( e_{mi}(t) \) vanish as \( t \to \infty \). Hence, (25) reduces to

\[
q_{md,i}(t) - q_{md,i}(0) \to q_i(t) - q_i(0) \tag{30}
\]

From Theorem 2 we also have that \( q_{md,i}(t) \to q_i(t) \to q_d \), which yields that \( q_i(t) \to q_d - q_{md,i}(0) + q_i(0) \) as \( t \to \infty \). Therefore, if \( q_{md,i}(0) = q_i(0) - \gamma_i \forall i \), we obtain that

\[
q_i(t) \to q_d + \gamma_i. \tag{31}
\]

Similarly, from (31) we have that

\[
\sum_{i=1}^{N} q_i(t) \to \sum_{i=1}^{N} q_d + \sum_{i=1}^{N} \gamma_i = \sum_{i=1}^{N} q_d, \text{ for } t \to \infty \tag{32}
\]

which finally yields that \( \frac{1}{N} \sum_{i=1}^{N} q_i(t) \to q_d \).

### 5 Examples

To illustrate the performance of the proposed control framework, we now present two teleoperation examples: one with a group of robotic manipulators and the other with a team of autonomous vehicles. Both examples are simulated using Matlab/Simulink platform.

#### 5.1 Coordination of a Group of Planar Robotic Manipulators

First, we consider a group of four, in-line, identical 2-DOF planar revolute-joint manipulators as shown in Figure 4. The nonlinear dynamic equations of motion for the manipulators are given according to (1) with

\[
M_i(q) = \begin{bmatrix}
\alpha_i & \beta_i \\
\beta_i & \gamma_i
\end{bmatrix}, \quad C_i(q, \dot{q}) = \begin{bmatrix}
\delta_i \ddot{q}^2_i & \delta_i (\dot{q}_1^i + \dot{q}_2^i) \\
-\delta_i \dot{q}^2_i & 0
\end{bmatrix}
\]

where \( q_1^i \in \mathbb{R} \) and \( q_2^i \in \mathbb{R} \) denote the angular position of the first and second links, respectively, and where \( \alpha_i = (0.834 + 0.150 \cos(q_2^i)) \text{ kg} \cdot \text{m}^2 \), \( \beta_i = (0.111 + 0.075 \cos(q_2^i)) \text{ kg} \cdot \text{m}^2 \), \( \gamma = 0.111 \text{ kg} \cdot \text{m}^2 \), and \( \delta_i = -0.075 \sin(q_2^i) \text{ kg} \cdot \text{m}^2 \).
(these system parameters correspond to the planar manipulators reported in [34]). The Rayleigh dissipation functions, which account for friction losses, are assumed to be given by $F_i(q_i) = 0.25q_i^Tq_i$, from which we can show that $ρ_i = 0.50 \text{ kg} \cdot \text{m}^2$. The four manipulators are further assumed to be subjected to nonuniform, constant coupling input delays listed as $T_{11} = 0.6 \text{ s}$, $T_{21} = 0.4 \text{ s}$, $T_{12} = 0.8 \text{ s}$, $T_{22} = 0.4 \text{ s}$, $T_{13} = 0.5 \text{ s}$, $T_{23} = 0.5 \text{ s}$, $T_{14} = 0.5 \text{ s}$, and $T_{24} = 0.7 \text{ s}$.

Having established the dynamics of the multi-agent system, we then design the reference model according to (7) and (8) with $A_m = -10I$ and $K_d = 20I$, where $I$ is the $2 \times 2$ identity matrix. The parameters of the scattering transformation are chosen as $b_i = ρ_i / 2 \forall i \in \{1, 2, 3, 4\}$, $K_{m1} = K_{m2} = 25I$, and $K_{m3} = K_{m4} = 15I$. Finally, the desired formation, depicted in Figure 4, is taken to be given by $q_d = [\pi/2, 0]^T \text{ rad}$, $γ_1 = −γ_4 = [0, \pi/4]^T \text{ rad}$, and $γ_2 = γ_3 = [0, 0]^T \text{ rad}$.

The response of the multi-agent system with the proposed MRRC framework is illustrated in Figures 5 and 6, where the manipulators are started from rest with configurations $q_1(0) = [−\pi, 0]^T \text{ rad}$, $q_2(0) = [0, \pi]^T \text{ rad}$, $q_3(0) = [0, \pi/2]^T \text{ rad}$, and $q_4(0) = [0, 0]^T \text{ rad}$, while the reference model is initialized in agreement with the conditions presented in Corollary 1. Note that all agents are able to converge to their desired configurations within the formation despite round-trip delays of up to 1.2 s. Interestingly, note also the presence of a right-shifted phase on the oscillation of the position error for the second, third, and four agents with respect to the phase of the first agent’s position error. This behavior can be attributed to a delayed effect on the motion of all agents caused by the larger formation error exhibited by the first agent. This observation also evidences the influence of one system’s behavior on the motion of all other agents.

Figure 4: Arrangement and desired formation of four planar manipulators.
Figure 5: Motion of planar manipulators. The vertical axes represent the Cartesian position \( y_i \) of the manipulators, where \( y_i = l \sin(q_1^i) + l \sin(q_1^i + q_2^i) \) and \( l = 0.8 \text{ m} \) denote the lengths of the first and second links. The base of the \( i \)th agent’s first link (illustrated in dark gray color) is anchored at \( y_i = 0 \text{ m} \). The position snapshots of the manipulators are time-spaced by 0.5 s.

Figure 6: Norm of the error between the \( i \)th agent’s position and its desired configuration.
5.2 Motion Control of a Team of Unmanned Vehicles

We now consider a group of six omnidirectional vehicles with nonlinear Lagrangian dynamics given by [42, 43]

\[
M_i \begin{bmatrix} \ddot{x}_i \\ \ddot{y}_i \\ \ddot{\phi}_i \end{bmatrix} + C_i(\dot{\phi}_i) \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\phi}_i \end{bmatrix} + N_i \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\phi}_i \end{bmatrix} = B_i(\phi_i)\hat{u}_i(t - T_i) \quad (33)
\]

where

\[
M_i = \begin{bmatrix} m_p + \frac{3I_r}{2r_i^2} & 0 & 0 \\ 0 & m_p + \frac{3I_r}{2r_i^2} & 0 \\ 0 & 0 & I_p + \frac{3I_rL_i^2}{r_i^2} \end{bmatrix}
\]

\[
C_i(\dot{\phi}_i) = \begin{bmatrix} 0 & 0 & 0 \\ \frac{3I_r}{2r_i^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

\[
N_i = \begin{bmatrix} m_p g \mu_1 & 0 & 0 \\ 0 & m_p g \mu_1 & 0 \\ 0 & 0 & m_p g \mu_2 \end{bmatrix}
\]

\[
B_i(\phi_i) = \begin{bmatrix} -\sin(\theta + \phi_i) & -\sin(\theta - \phi_i) & \cos \phi_i \\ \cos(\theta + \phi_i) & -\cos(\theta - \phi_i) & \sin \phi_i \\ L_i & L_i & L_i \end{bmatrix}
\]

We further assume that \( \phi_i(t) \) is a slow time-varying state, such that

\[
\hat{u}_i(t) = B(\phi_i(t))B(\phi_i(t - T_i))^{-1}u_i(t - T_i) \approx u_i(t - T_i).
\]

Then, (33) conforms to (1) with

\[
F_i(\dot{q}_i) = \frac{1}{2}q_i^T N_i q_i.
\]

The system parameters are \( I_r = 0.52 \text{ kg} \cdot \text{m}^2 \), \( I_p = 0.17 \text{ kg} \cdot \text{m}^2 \), \( g = 9.81 \text{ kg} \cdot \text{m/s}^2 \), \( \mu_1 = 0.25 \), \( \mu_2 = 0.1 \), and \( \theta = 0 \text{ rad} \) for all vehicles; \( m_{p_i} = 9.58 \text{ kg} \), \( r_i = 0.079 \text{ m} \), and \( L_i = 0.205 \text{ m} \) for the first, second, and third vehicles; and \( m_{p_i} = 11.58 \text{ kg} \), \( r_i = 0.089 \text{ m} \), and \( L_i = 0.305 \text{ m} \) for the fourth, fifth, and sixth vehicles. The reader can verify that (2) is satisfied for all agents with \( \rho_i = 9.40 \).

The reference model and the scattering transformation parameters are finally chosen as

\[
A_m = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix}, \quad K_d = \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 2 \end{bmatrix}
\]

\[
b_i = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad K_{m_i} = \begin{bmatrix} 250 & 0 & 0 \\ 0 & 250 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]
∀i. The round-trip coupling delays between controller and vehicles are taken to fluctuate between 1.5-1.7 s (i.e., $T_{11} = 1.0$ s, $T_{21} = 0.5$ s, $T_{12} = 1.2$ s, $T_{22} = 0.5$ s, $T_{13} = 0.8$ s, $T_{23} = 0.8$ s, $T_{14} = 0.8$ s, $T_{24} = 0.7$ s, $T_{15} = 0.5$ s, $T_{25} = 1.0$ s, $T_{16} = 1.2$ s, and $T_{26} = 0.4$ s).

In this example, the agents are commanded to converge to a trapezoid-like formation with vertices $\gamma_1 = [1 \text{ m}, 1 \text{ m}, 0 \text{ rad}]^T$, $\gamma_2 = [0 \text{ m}, 2 \text{ m}, 0 \text{ rad}]^T$, $\gamma_3 = [-1 \text{ m}, 3 \text{ m}, 0 \text{ rad}]^T$, $\gamma_4 = [-1 \text{ m}, -3 \text{ m}, 0 \text{ rad}]^T$, $\gamma_5 = [0 \text{ m}, -2 \text{ m}, 0 \text{ rad}]^T$, and $\gamma_6 = [1 \text{ m}, -1 \text{ m}, 0 \text{ rad}]^T$. Simultaneously, the formation centroid is commanded to track a time-varying trajectory given by $q_d(t) = [x_d(t), y_d, \phi_d]^T = [t/5, 0, 0]^T$. The response of the multi-vehicle system is illustrated in Figure 8 where the agents are started from rest with configurations $q_1(0) = [-3 \text{ m}, 6 \text{ m}, 0 \text{ rad}]^T$, $q_2(0) = [-6 \text{ m}, 6 \text{ m}, \pi/2 \text{ rad}]^T$, $q_3(0) = [-6 \text{ m}, -2 \text{ m}, -\pi \text{ rad}]^T$, $q_4(0) = [-6 \text{ m}, -3 \text{ m}, 0 \text{ rad}]^T$, $q_5(0) = [-1 \text{ m}, -6 \text{ m}, \pi/4 \text{ rad}]^T$, and $q_6(0) = [-4 \text{ m}, 0 \text{ m}, -\pi/4 \text{ rad}]^T$. The reference model is initialized according to Corollary 1. Note that the agents are able to satisfactorily follow the desired trajectory (traced in the plot by the dotted line) while keeping the trapezoid-like formation despite round-trip delays of up to 1.7 s. The norm of the position error and the heading angle error for all vehicles are also illustrated in Figures 9 and 10 respectively. Observe that the vehicles’ heading error converges to zero, whereas the position error converges to a nonzero value. The nonzero constant error in Figure 9 is due to the fact the desired trajectory (i.e., $x_d(t)$) does not converge to a static configuration but diverges at a constant rate (the reference information arrives delayed to the the agents).

Figure 7: Omnidirectional vehicle.
Figure 8: Cartesian motion of omnidirectional vehicles for $t \in [0 \text{ s}, 100 \text{ s}]$. The position in $x$ and $y$ for the first, second, and third agents is denoted by the light-gray colored circles, while the position for the fourth, fifth, and sixth agents is indicated by the dark-gray colored circles. Each position mark is time-spaced by 1.5 s and newer data is imposed over older data. The heading angle of each vehicle (i.e., $\phi$) is indicated by the direction of the triangle imprinted within the circular marks. The dotted line at the center of the plot traces the desired trajectory of the formation’s centroid. The open trapezoid-like shape indicates the desired formation and desired agents’ position at $t = 100 \text{ s}$.

Figure 9: Norm of the position error, i.e.,
$$
\| [x_i(t) - x_d(t) - \gamma_1^i, y_i(t) - y_d - \gamma_2^i] \|. $$
6 Conclusions

In this paper, we have proposed and studied the use of a passivity-based MRRC framework for the remote control of $N$ nonlinear Lagrangian systems with nonuniform control input delays. The proposed control framework extends the control architecture introduced in [35] for single, time-delay robotics systems to the general case of multiple nonlinear agents. We showed that the proposed controller, comprised of a linear reference model and $N$ scattering transformation blocks, guarantees the stability, motion coordination, and formation convergence of a group of output strictly passive systems with any large constant input delays using concepts of passivity and Lyapunov stability. Furthermore, we showed that the design control parameters are delay-independent and that the overall controller is robust to model uncertainties, only requiring the knowledge of a lower bound on the dissipation rate $\rho_i$. Along with the theoretical analysis, two numerical examples with planar manipulators and unmanned vehicles were presented to validate the proposed MRRC framework. To the best of our knowledge, this paper constitutes the first time that the concept of passivity-based scattering transformation is employed in the stabilization and control of multiple nonlinear systems with control input delays and one of the first research efforts to address the coordination of nonlinear systems with pure delayed control. Research extensions to the ideas presented in this paper include the stability analysis of the overall system under time-varying delays and data transmission losses, the compensation of initial position errors, and the addition of direct inter-agent communication.

References


