Probability Density Analyses of Partially Coherent Gaussian Laser Beam in a Turbulent Environment

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Abstract

The present study aims to find how power distribution using various base probability density functions and window size affects the statistical properties of a laser after passing through a turbulent environment. The collected data varied in environment as well as beam polarization. A partially coherent beam was propagated through a controlled turbulent environment in which temperature and air velocities were varied. The results are provided as a graph and table.

1. **Introduction**:

With the increased need of Line of Sight (LOS) communications utilizing laser light to establish secure communications, the environmental effects of differing atmospheric conditions on a propagating beam must be better understood. Various to atmospheric conditions could impart unwanted frequencies that could affect signal clarity. Additionally, by determining the probability density of the beam will allow for an understanding of optimal conditions for utilizing the communications. The probability density of the beam's intensity can also forecast the beams consistency across time. There are several different methods available to minimize signal loss and interference, including: multiple transmitter apertures, adaptively controlling the transmitter beam size and utilizing adaptive optics and channel coding. [1]

The present study aims to find the probability density function of polarized Gaussian beams when passing through a turbulent environment. The beam is made partially coherent by reflecting it off a Spatial Light Modulator (SLM) providing the ability to manipulate the phase modulation experienced by the beam. The turbulence diminishes the intensity creating a distorted and weakened signal. The beam propagates through a controlled turbulent environment to simulate a variety of natural and artificial disturbances such as moving weather fronts or exhaust from a mechanical part. Two heat guns in orthogonal directions generate the simulated turbulence in a controlled chamber, which allows for various turbulence effects. The two main types of interference which can affect a laser beam are scattering, the effect of particulates interfering with beam propagation and turbulence, the effect of thermal and pressure gradients effecting beam propagation. This experiment only addresses turbulence. The collected data is then processed to determine a probability density function. The primary goal is to determine possible statistical properties based on the manipulation of the beams PDF.

Possible benefits from determining the optimal polarization for beam propagation in a turbulent environment include improved signal clarity for LOS communications and a more predictable assessment of the reliability of LOS communications. Understanding the combination of these effects with respect to polarization could provide an understanding of the best conditions and manipulations to be utilized for beam propagation.

2. Kernel Method

The Experiment aims to generate a probability density function (PDF) for the power distribution of a laser beam. A PDF allows one to predict the likelihood of an event occurring. A distribution of

the data according to a function *f*, probability of the likelihood of it occurring between two events a and b can be expressed as:

$$P(a < X < b) = \int_a^b f(d)dx \quad \text{for all } a < b.$$

This allows for an estimate of an event occurring known as density estimate. There are many different methods for estimating data density. The oldest, widest used, and most basic is the histogram which plots a number of bins versus the frequency of occurrence. The histogram can be described by the equation:

$$\hat{f}(x) = \frac{1}{n} \times \frac{(\text{no. of } X_i \text{ in the same bin as } x)}{(\text{width of bin containing } x)}.$$

The Histogram provides a base to analyze the data but has many shortcomings due to the discontinuity of a histogram making it difficult to interpolate the data between the bin widths.

In this experiment I will use the Kernel Method for data estimation as developed by Silverman. The PDF *K* must fulfill the following equation [2]:

$$\int_{-\infty}^{\infty} K(x)dx = 1.$$

The Kernel estimator generates the PDF by summing the estimator function for each data point at its distance from the origin. The PDF is defined by the equation [2]:

$$\text{PDF} = \frac{1}{nh} \sum_{i=1}^{n} K(\frac{x - X_i}{h})|$$

Where *h* is the window width also known as the smoothing parameter and X*i* denotes the *i*-th element of the data array. This method is similar to a histogram where the vale is summed at a location, but instead of a discontinuous step function it sums functions of the shape K which is determined by estimation equation used for K. The *h* value determines the width of each data estimation point. Silverman provides the Fig. 1 to demonstrate the effect of changing the window width *h* on 7 pseudo-random generated points [2].

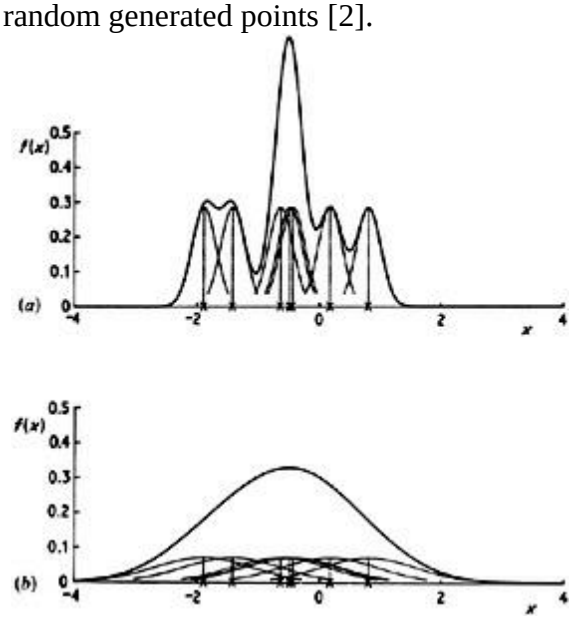


Fig.1. Kernel estimates showing individual kernels. Window widths: (a) 0.2; (b) 0.8. [2]

As observed a *h* value that is too small provides a function of spikes close to a sum of Dirac-delta functions and generates false data distributions and a *h* value that is too large eliminates nearly all data facets [2]. The Chosen function K which the PDF is modeled on must also be a data density function. The method used does have a drawback that the window width is constant throughout so any data with a bump in the tail would be obscured. However, this method will be sufficient as turbulence will not generate a bump in the tail which is critical to understanding the power density of the laser at the target.
The Density estimation function K that I will use will be based on three common PDFs.
The normal Gaussian distribution curve where μ=0 and σ=1.

$$K(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{(x-\mu)^2}{2\sigma^2}),$$

The Cauchy distribution where x_0=0 and γ=1.

$$K(x) = \frac{1}{\pi\gamma\left[1+\left(\frac{x-x_0}{\gamma}\right)^2\right]}$$

And the Laplace distribution where μ=0 and b=1.

$$K(x) = \frac{1}{2b}\exp\left(-\frac{|x-\mu|}{b}\right)$$

3. Experimental Set-up

The experiments assessed the probability density distribution of various environmental conditions to analyze the consistency of the target power. Each experiment was carried out over 1024 independent readings at a sampling time of 30 micro seconds to generate a large data pool.

See appendix A

4. Probability Density Function Method

The PDF were generated using a MATLAB function provided by Professor Reza Malek-Madani. The Function has three inputs: the data, the window width (*h*), and the x value. In this experiment the x value covers the spectrum of power down to the measured values of power.

PDF Algorithm:

- For loop (repeat for every value in the data array)
  - Compute the term according to the Kernel method
  - Compute the individual probability density function at each point using one of the base PDFs
  - Add the new PDF to the previous PDF to generate the sum via the Kernel method

See appendix B for MATLAB functions

5. Results

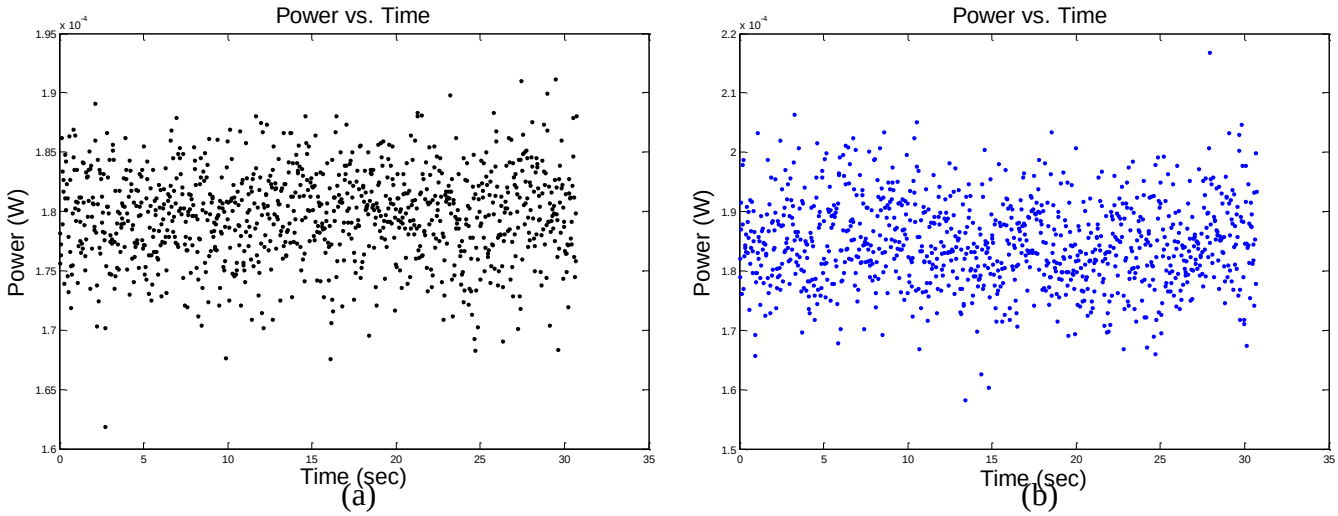


Fig. 2. Laser beam power at sensor varying over time passing through a controlled turbulent environment. The horizontal heat guns outputting at 400 °F and at the high speed and the vertical heat guns outputting at 200 °F and at the base speed. (a)vertically polarized laser and (b) horizontally polarized laser.

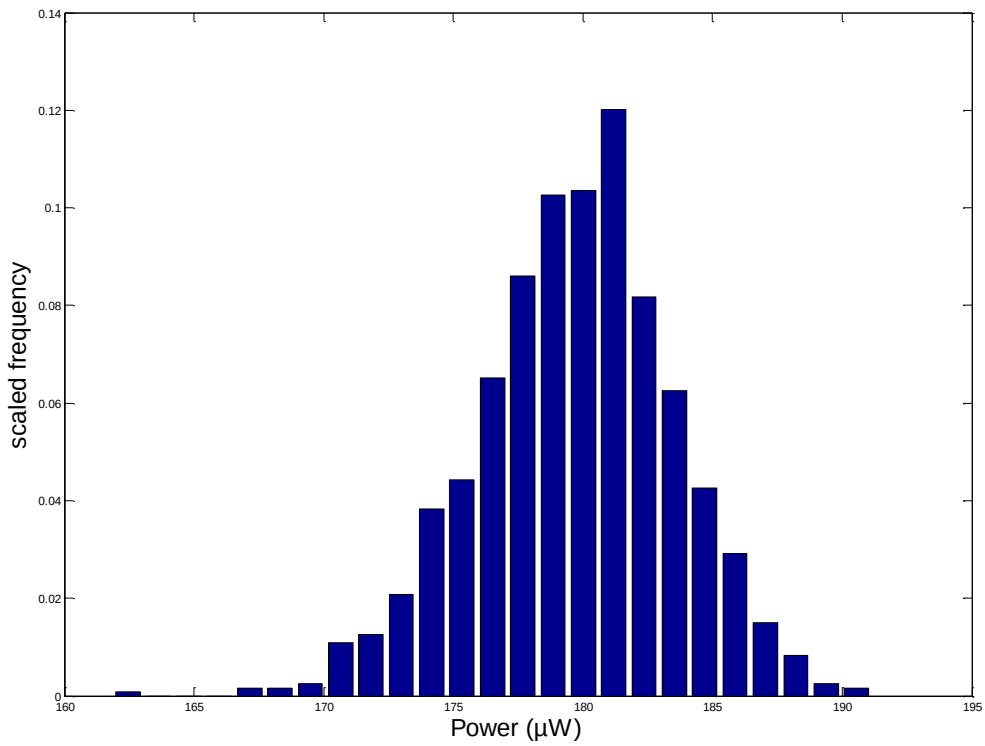


Fig.3. Scaled frequency histogram of the data collected in Fig1.a.

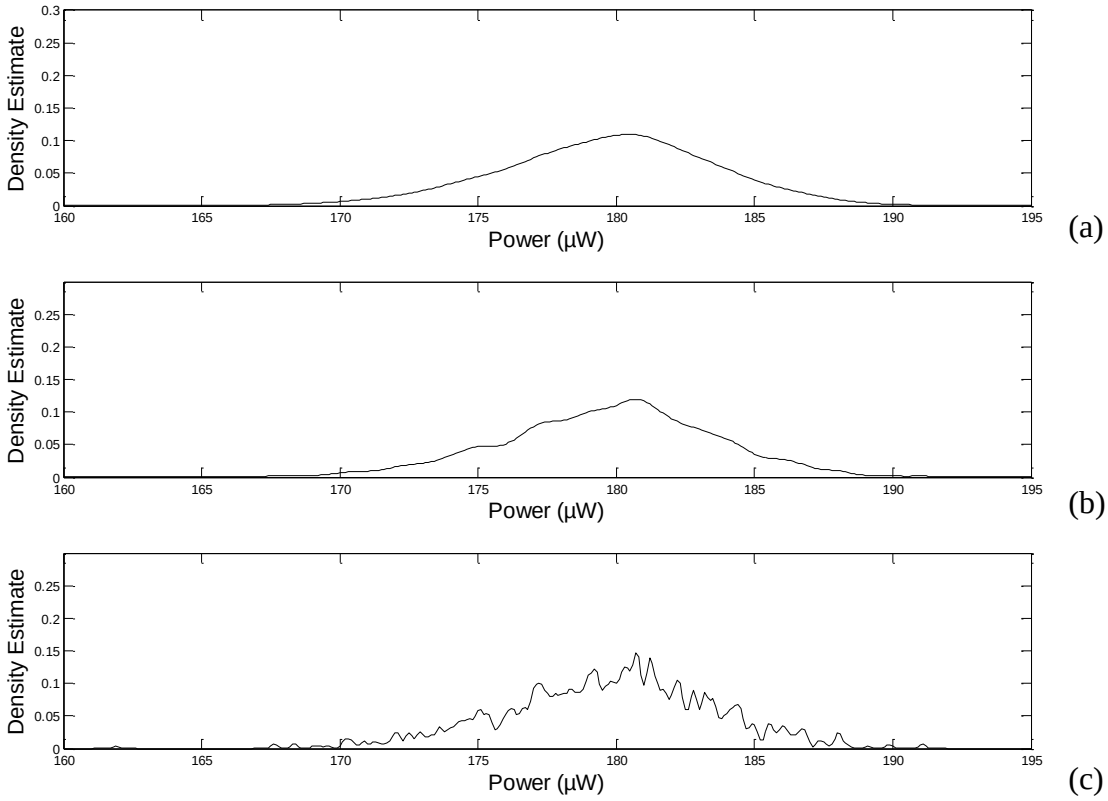


Fig.4. Probability density function from data collected in Fig1.a. The PDF uses a normal Gaussian distribution as the K function. Window widths: (a) 1; (b) 0.5; (c) 0.1. Density estimate scaled by 10.

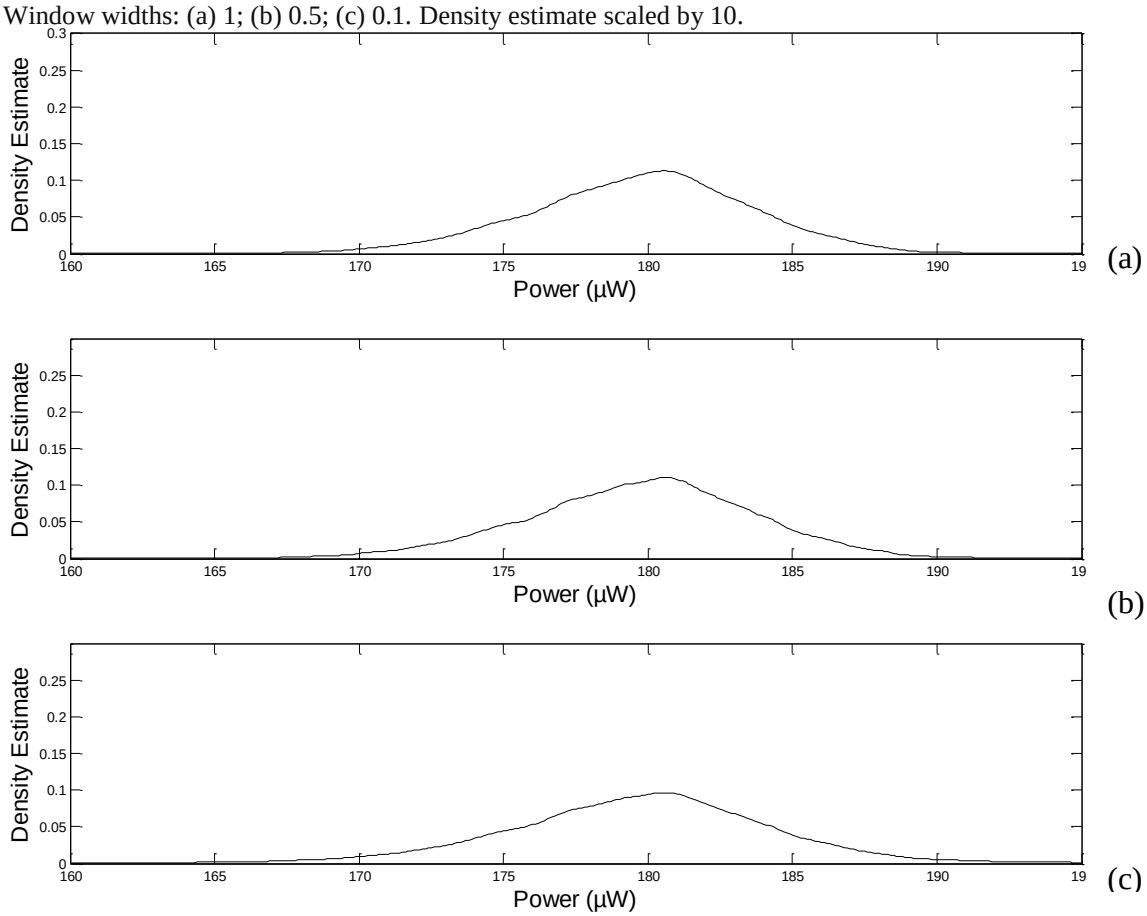


Fig.5. Probability density function from data collected in Fig1.a. The PDF uses a window width of 0.8. K functions: (a) normal Gaussian distribution; (b) Laplace distribution; (c) Cauchy distribution. Density estimate scaled by 10.

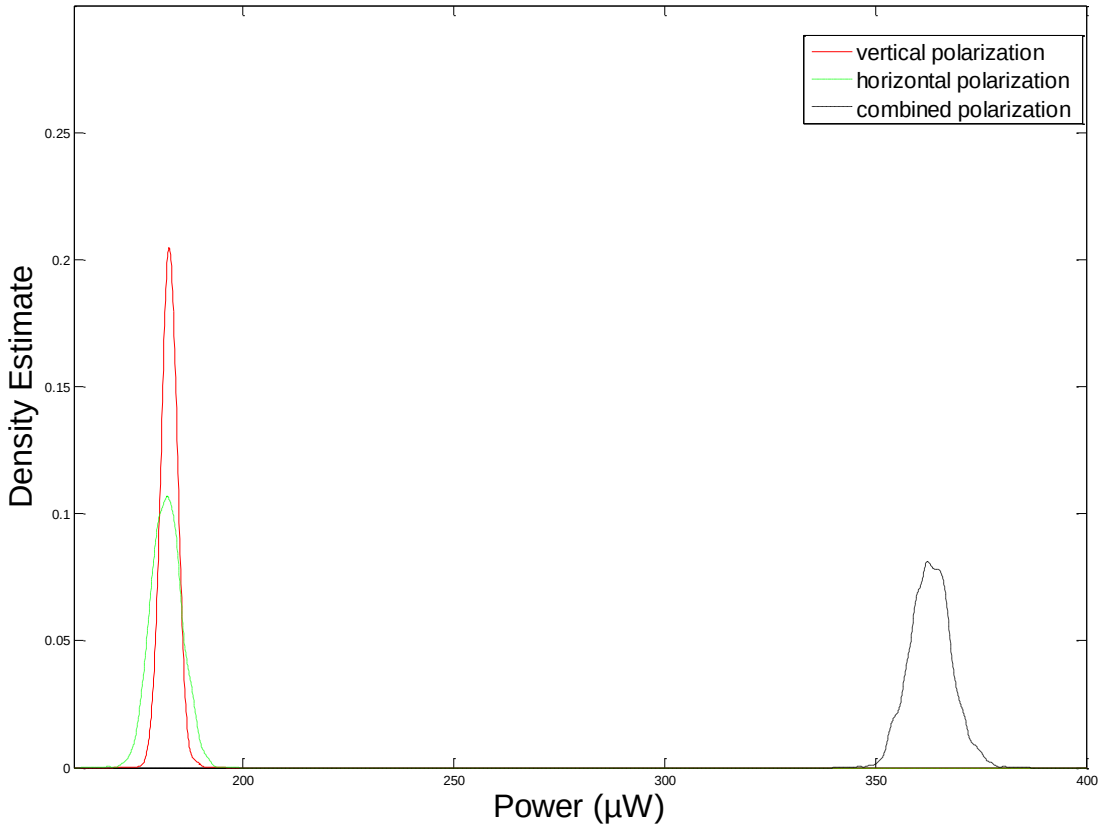
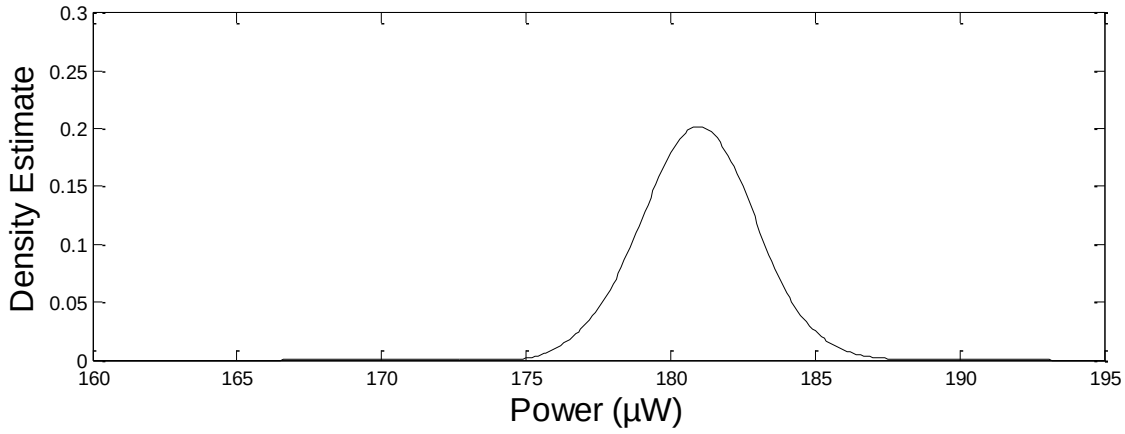
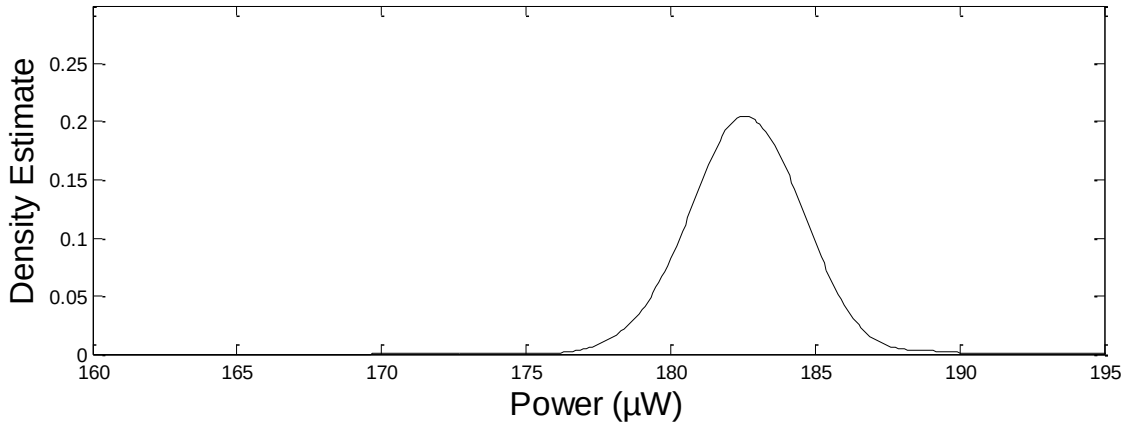


Fig.6. Comparison of the probability density function for vertical, horizontal, and combined polarization laser beam passing through a controlled turbulent environment. The horizontal heat guns outputting at 200 °F and at the high speed and the vertical heat guns outputting at 800 °F and at the base speed. Probability density function using window width 0.8 and using a normal Gaussian distribution as the K function. Density estimate scaled by 10.

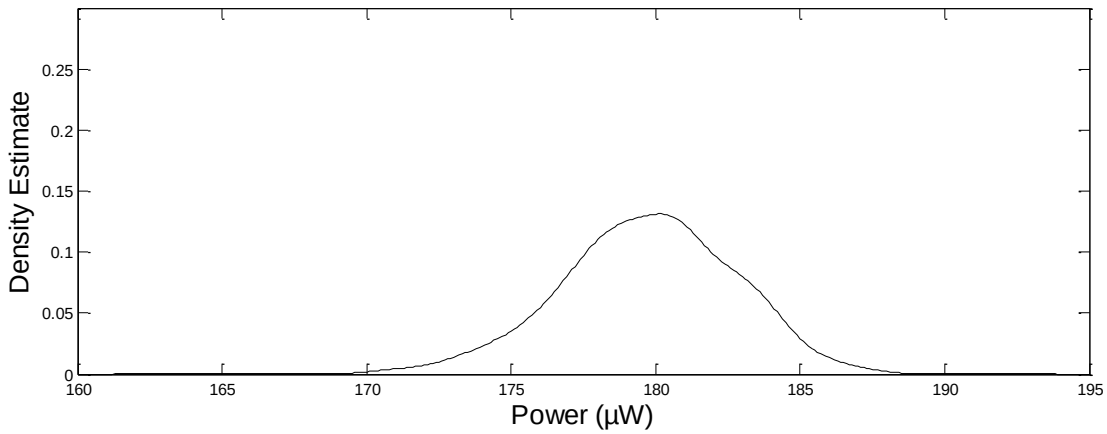


(a)

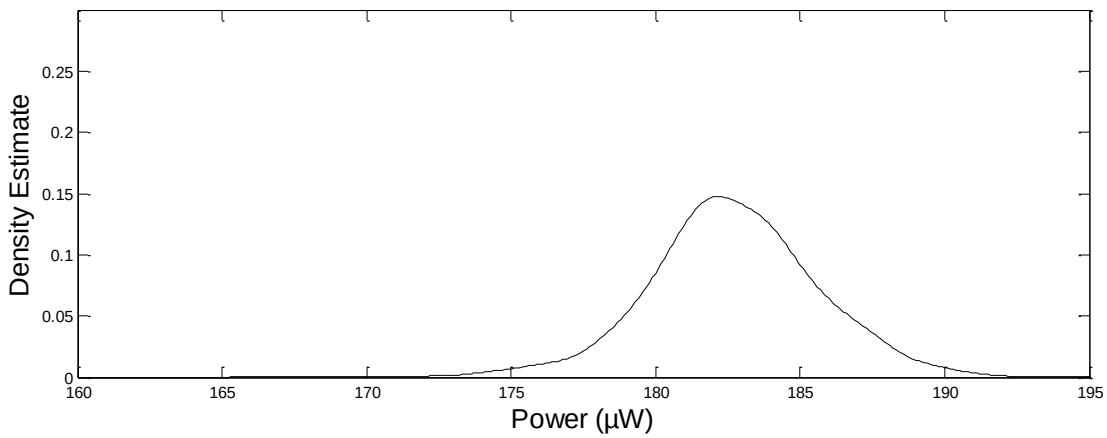


(b)

Fig.7. Comparison of the probability density function for a vertically polarization laser beam passing through a controlled turbulent environment at two different temperatures. The horizontal heat guns outputting at 200 °F and at the high speed and the vertical heat guns outputting at (a) 400 °F (b) 800 °F and at the base speed. Probability density function using window width 0.8 and using a normal Gaussian distribution as the K function. Density estimate scaled by 10.



(a)



(b)

Fig.8. Comparison of the probability density function for a vertically polarization laser beam passing through a controlled turbulent environment at two speed orientations. (a) The horizontal heat guns outputting at 200 °F and at the high speed and the vertical heat guns outputting at 200 °F and at the base speed. (b) The horizontal heat guns outputting at 200 °F and at the base speed and the vertical heat guns outputting at 200 °F and at the high speed. Probability density function using window width 0.8 and using a normal Gaussian distribution as the K function. Density estimate scaled by 10.

Scintillation Index over 1024 measurements			
Heat gun setting	**Horizontal**	**Combined**	**Vertical**
Fast Horizontal 400°F Vertical one off	1.52E-03	8.65E-04	5.00E-04
Fast Horizontal	5.41E-04	2.92E-04	2.76E-04
Fast Vertical	7.79E-04	5.74E-04	2.53E-04
Fast Horizontal 400°F Vertical	5.62E-04	2.33E-04	1.04E-04
Fast Horizontal 800°F Vertical	3.87E-04	1.81E-04	9.99E-05

Table 1. The Scintillation Index of collected data from various environments and beam polarizations. The heat gun column provides information on any change in setting from the base setting of 200 °F and at the base speed.

Summary:

The experiment's goal was to assess the effect of beam polarization and environmental turbulence of a partially coherent Gaussian beam on the probability density function (PDF). The data analysis included changing the window width as well as the K function, the base PDF that each data point is overlaid with to generate the beam's PDF. The experiment included splitting Gaussian beam into two independent branches, and then simultaneously modulating the beam out of phase to produce a partially coherent beam using computer controlled SLMs. The beam was polarized and either recombined with the perpendicular polarization or left as a single polarized beam and then passed through a controlled turbulent environment. The beam's power was then recorded by a polarimeter to record the power at the target.

Through interpolating the changes in Fig. 3 the optimal window width appears to be 0.8 as statistical properties are not obscured nor does the function possess erroneous spikes that could lead to false data analysis. Fig. 5 demonstrates the effect of changing the K function on PDF. The Laplace and Cauchy distribution functions provide a PDF with a longer tail. The results of Fig. 6 demonstrated that a vertically polarized beam produces a narrower PDF than the horizontal or combined beam at termination when traveling through a multitude of turbulent environments. Additionally, Fig. 7 demonstrates that an increase in temperature lessens the effect of power loss on a vertically polarized beam by a slight amount. The results of Fig. 8 indicate that a vertically polarized partially coherent beam passing through an environment with greater turbulence parallel to the polarization generates a smother PDF as opposed to an increase in perpendicular turbulence. This provides immense benefits for secure communications as it would allow for minimal data and power loss while ensuring that the information is not intercepted.

References

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Yubo Sheng, Baoyu Zheng, "Turbulence Mitigation Scheme for Optical Communications using Orbital Angular Momentum Multiplexing Based on Channel Coding and Wavefront Correction," arXiv:1401.7558 [physics.optics]

[2] B.W. Silverman, "Density Estimation For Statistics And Data Analysis," Volume 26 of Chapman & Hall/CRC Monographs on Statistics & Applied Probability.

[3] Encyclopedia Britannica: optics: Mutual coherence function.
http://www.britannica.com/EBchecked/topic/430511/optics/37991/The-mutual-coherence-function.

[4] Shengmei Zhao, Bei Wang, Li Zhou, Longyan Gong, Wenwen Cheng,
Yubo Sheng, Baoyu Zheng, "Turbulence Mitigation Scheme for Optical Communications using Orbital Angular Momentum Multiplexing Based on Channel Coding and Wavefront Correction," arXiv:1401.7558 [physics.optics]

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Appendix

A:
Based on the experimental setup provided by Professor Svetlana Avramov-Zamurovic.
A coherent 2 mW, He-Ne laser (A) was used to generate a Gaussian beam with a diameter of 0.5 mm with a 632.8nm wavelength (see Figure 1). A beam expander (B) was used to expand the beam to fill the spatial light modulator window. The expanded beam was split with a 50/50 beam splitter (C) to generate vertically and horizontally polarized branches. A half-wave plate (D) rotated the beam generated by laser (A). The SLM (E) was used to modulate the phase of the beams at 90.9 Hz. to pseudo-randomize the phase, which resulted in a partially coherent beam. A neutral density filter (F) adjusted the power level both the horizontal and vertical branch have equal power. A linear polarizer (G) was used to polarize the beams and the mirror (H) redirected the beam to be recombined if needed. The vertically and horizontally polarized beams could be combined at a 50/50 Pellicle beam splitter (I) to allow for a combined or single polarized beam. The beam then passed through a mechanical iris (J) to reduce diffraction effects. The beam then passed through the turbulence emulator (L), a tunnel of controlled turbulent environment, used four heat guns with various speed and temperature with two oriented vertically and two horizontally orthogonal to the beam. The beam terminates and was captured by a polarimeter (M) that records the average power every 0.3 seconds and relays the data to a computer (K).

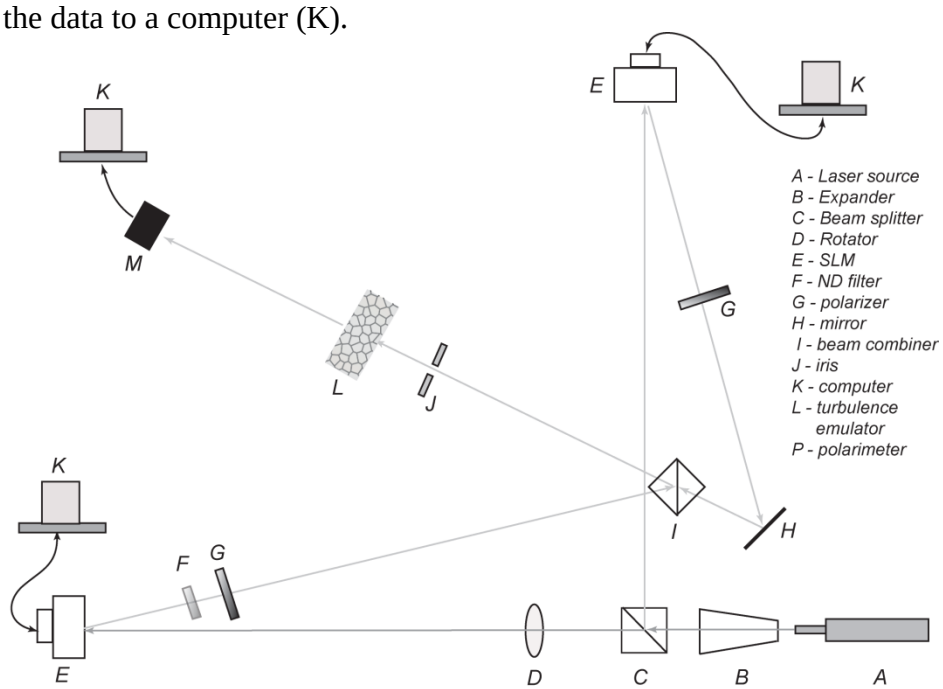
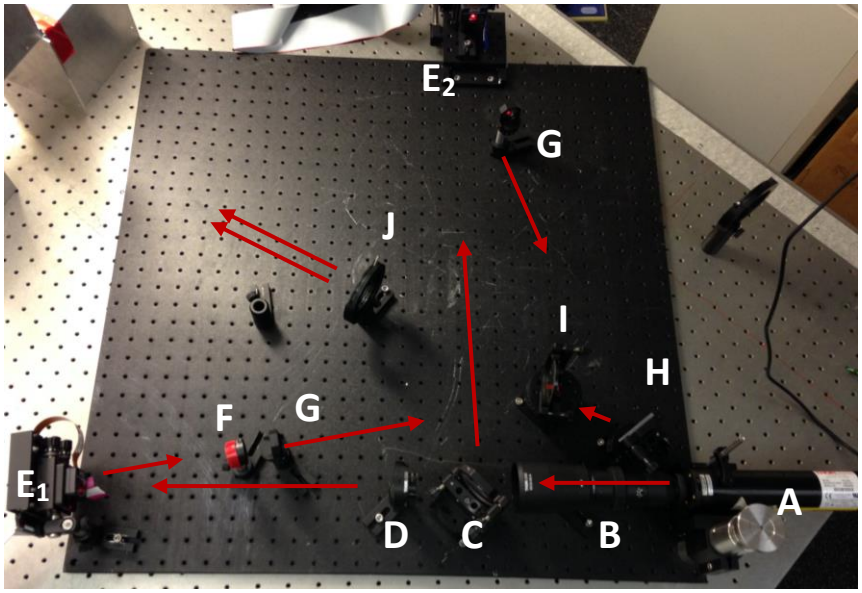
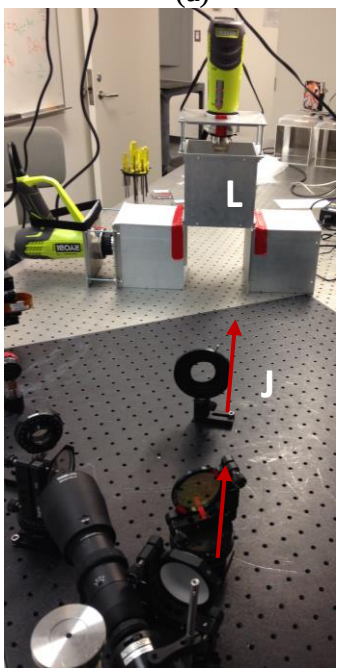


Fig. 1. Experimental setup for generating a pseudo-random partially coherent beam.



(a)



(b)

Fig. 2. Photographs of (a) the beam randomizer; (b) the turbulence emulator.

B:

1. Normal distribution

```
function PDF=KernelMethodnormal(DATA,x,h)
nn=length(DATA); %number of data points
sum=0;
for i=1:length(DATA) %use a for loop to sum the K function
term=((x-DATA(i))/h).^2; %input term according to Kernel method
sum=sum+1/nn/h*1/sqrt(2*pi)*exp(-term/2); %Normal distribution function
end
PDF=sum; %set the sum of the K functions equal to the PDF
end
```

2. Laplace distribution function

```
function PDF=KernelMethodLaplace (DATA,x,h)
nn=length(DATA);%number of data points
sum=0;
for i=1:length(DATA)%use a for loop to sum the K function
term=((x-DATA(i))/h); %input term according to Kernel method
sum=sum+1/nn/h*1*exp(-abs(term)/1);%Laplace distribution function
end
PDF=sum;%set the sum of the K functions equal to the PDF
end
```

3. Cauchy distribution function

```
function PDF=KernelMethodCauchy (DATA,x,h)
nn=length(DATA);%number of data points
sum=0;
for i=1:length(DATA)%use a for loop to sum the K function
term=((x-DATA(i))/h).^2; %input term according to Kernel method
sum=sum+1./nn./h*1./(pi*(1+term)); %Cauchy distribution function
end
PDF=sum; %set the sum of the K functions equal to the PDF
end
```