Mathematical Modeling of Airy Disk Diffraction

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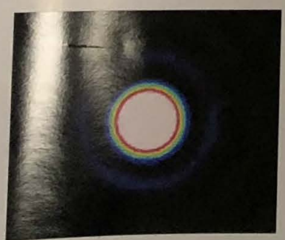
Abstract

Light traveling through a medium does not display uniform characteristics throughout the length of the beam, but rather the intensity of light varies depending on the distance from the origin, or aperture, of the light. This experiment is an attempt to describe the behavior of light intensity both theoretically and practically.

Background

As light passes through or across an appropriately sized aperture or obstruction, Airy disk diffraction patterns appear as rings around a central disk of light. These rings are a result of the constructive and destructive convergence of light, demonstrating light's wave characteristics.



These constructive and destructive patterns corresponds with varying intensity. This characteristic can be modeled mathematically.

Methods

By integrating the solution to the wave equation, we were able to obtain an ideal intensity function that was then manipulated to account for distance. By using Mathematica, we were then able to generate 2D and 3D graphs of the normalized intensity. We then compared these results to actual data results.

Having established the normalized intensity, we then introduced an obstruction, i.e. a rotating fan, into the system, thus producing an oscillating graph of intensity on an oscilloscope. The slope of this graph could then be used to determine the location of the beam waist.

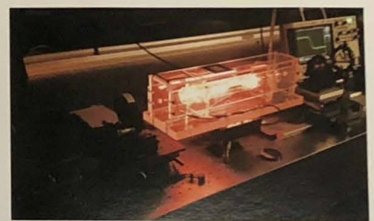
$$I(x,y,z) = \frac{1}{\Theta_0^2+\Lambda_0^2}\exp\left(-\frac{-2(x^2+y^2)}{W(z)^2}\right)$$

$$\Theta_0 = 1 - \frac{z}{F_0}$$

$$\Lambda_0 = \frac{2z}{kW_0^2}$$

$$W(z) = W_0\sqrt{\Theta_0^2+\Lambda_0^2}$$

$$p(\Omega, z) = \int\int_\Omega I(u,v,z)dA.$$



Results

```
lambda = 633*10^-9;
k = 2 Pi / lambda;
R = 10^7*lambda;
** Defining variables in integrating e^ikr **
  NewIntensity[X_, Y_] :=
 1/R^2
  Abs[NIntegrate[r Exp[-I*k/R (X*r*Cos[theta] + Y*r*Sin[theta])],
     {r, 0, 1000*lambda}, {theta, 0, 2 Pi}]]^2
** Integrating e^ikr **

NewIntensity[10*lambda, 10*lambda]
3.95456×10^-14

Plot3D[NewIntensity[X, Y], {X, -10000*lambda, 10000*lambda},
 {Y, -10000*lambda, 10000*lambda}]
** Generating plot of intensity given conditions on X and Y **
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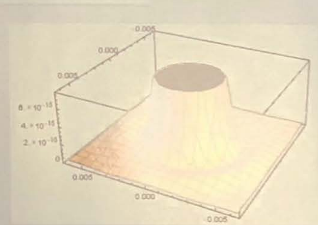
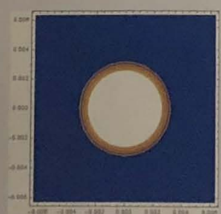


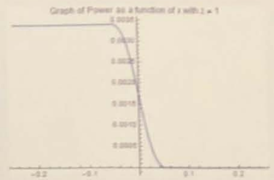
Mathematica model of beam intensity

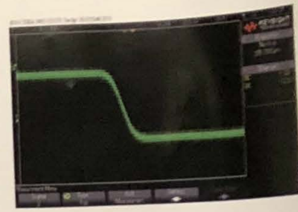


Left: 2D model of beam intensity showing beam size radius

3D beam intensity from experimental setup

Below: A Mathematica built representation of an obstruction pattern





Above: An actual representation of an obstruction pattern from an oscilloscope

Conclusion

Comparing the mathematical model generated in Mathematica by integrating e^ikr over the disk radius indicates that the model is very similar to the experimentally obtained Airy disk. Likewise, the modeling of an obstruction also had similarities to the experimental results

References

Hecht, Eugene. *Optics*. 2nd Edition. 454-487.

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